THE EFFECT OF ENDOGENOUS TIMING ON COORDINATION UNDER ASYMMETRIC INFORMATION: AN EXPERIMENTAL STUDY[*]

FRANCESCO BRINDISI [†]
NYCEDC AND SIPA

BOĞAÇHAN ÇELEN [‡]
COLUMBIA UNIVERSITY

KYLE HYNDMAN [§]
SOUTHERN METHODIST UNIVERSITY

MAY 16, 2011

ABSTRACT

This paper investigates the role of endogenous timing of decisions on coordination under asymmetric information. In the equilibrium of a global coordination game, where players choose the timing of their decision, a player who has sufficiently high beliefs about the state of the economy undertakes an investment without resort to any delay. This decision triggers an investment by the other player whose beliefs would have led to inaction otherwise. Endogenous timing has two distinct effects on coordination: a learning effect (early decisions reveal information) and a complementarity effect (early decisions eliminate strategic uncertainty). We also show that endogenous timing enhances ex ante welfare compared to the simultaneous case. The experiments we conduct to test these results show that the learning effect of timing has more impact on the subjects' behavior than the complementarity effect. We also observe that subjects' welfare improves significantly under endogenous timing.

JEL CLASSIFICATION NUMBERS: C72, C91, D62, D82, D83.

KEYWORDS: Coordination, delay, information, global games, experiment.

---

[*] We gratefully acknowledge helpful discussions with Alessandro Citanna, John Duffy, Stephen Morris, Hyun Song Shin, and Paolo Siconolfi, as well as seminar participants at New York University, University of Maryland, Rutgers University, Columbia University, Fordham University, University of Texas at Austin, Boston College, Carnegie Mellon University, University of British Columbia, McGill University, Yeshiva University and Georgetown University. This paper has also benefited from suggestions by the participants of the AEA 2009 Annual Meeting and the 2009 ESA North American Conference.

[†] V.P. Research and Analysis Department, NYCEDC, 110 William Street, New York, NY 10038. E-mail: fb2102@columbia.edu.

[‡] Graduate School of Business, Columbia University, 3022 Broadway, 602 Uris Hall, New York, NY 10027. E-mail: bc2132@columbia.edu, url: http://celen.gsb.columbia.edu.

[§] Department of Economics, Southern Methodist University, 3300 Dyer Street, Suite 301R, Dallas, TX 75275. E-mail: hyndman@smu.edu, url: http://faculty.smu.edu/hyndman.

1 INTRODUCTION

There are many economic activities that are characterized by strategic complementarities where individuals can achieve a desirable outcome if they coordinate their strategies. Such activities include the agglomeration of businesses (Caplin and Leahy [14]), technology adoption (Katz and Shapiro [40, 41]), crop choice by subsistence farmers (Conley and Udry [16] and Engle-Warnick and Laszlo [23]), bank runs and currency attacks (Diamond and Dybvig [20], Morris and Shin [46] and Goldstein and Pauzner [33]), and foreign direct investment (FDI) (Jordaan [38]), among many others. In this paper we frame our discussion around the FDI example. Indeed, in the international trade literature it has often been argued that the probability of a FDI project's success increases if and only if the flow of FDI into the host market is high, because the presence of a large number of FDI projects can create positive externalities.

Another common characteristic of the examples mentioned above is the existence of asymmetric information. For instance, potential investors in foreign markets have myriad sources of information regarding the uncertainty that their decision involves. Uncertainty in FDI decisions can include political uncertainty (Rodrick [49], Alesina and Perotti [2], and Busse and Hefeker [8]), demand uncertainty (Goldberg and Kolstad [32], Aizenman and Marion [3]), exchange rate uncertainty (Cushman [9], Schmidt and Broll [50]), and cost uncertainty (Creane and Miyagiwa [15]). In many cases, individuals acquire private information from different sources regarding these uncertain events. For example, as Markusen [44] argues, "[... a] multinational firm may adopt some contractual agreement with a local agent as a means of exploiting any superior information the agent may possess regarding market characteristics." As a result, each firm's information is determined by the contractual relationship with a local agent. This can create asymmetry because potential investors will pair with different local agents and will, therefore, have access to different sources of information.

The effect of strategic timing has been subject to analysis both under pure information externality (Chamley and Gale [11], Gul and Lundholm [34]) and under strategic complementarities (Bolton and Farrell [7], Farrell [24], Farrell and Saloner [25, 27]). Under pure information externality, time serves as a vehicle for disseminating information between individuals who make their decisions at different moments. We will refer to this effect of time as the *learning effect*. Where strategic complementarities exist, in our analysis of the timing of decisions we focus on the role of timing in enabling coordination between individuals. In the sequel, we will refer to this effect as the *complementarity effect* of strategic timing.

In the FDI example referred to above, when there are many FDI projects there will be higher returns on new projects because of strategic complementarities. A dearth of existing FDI projects, on the other hand, in turn leads to stagnation because of strategic complementarities. In these cases, the complementarity effect of timing is evident. Suppose that a critical number of firms make an early decision to undertake an FDI project. The mere awareness of these projects eliminates the risk of coordination failure for other firms and leads them to undertake similar projects because of the existence of complementarities.

The outcome of economic activities can also be affected by the timing of when information is revealed by individuals' decisions. A firm that undertakes a FDI project reveals valuable information about the profitability of the project. Early investors therefore trigger a process through which valuable information aggregates. At the same time, the option of having access to this information in later periods (i.e., the learning effect) creates an incentive for the investors to wait and observe others' decisions. In fact, as Chamley and Gale [11] show, if the incentive to delay the investment is sufficiently strong, no investment is undertaken even though it is beneficial for all investors.

Despite the substantial amount of research analyzing the complementarity and learning effects separately, there is little theoretical literature (viz. Dasgupta [19], Heidhues and Melissas [36]) that studies the two effects together. Furthermore, to the best of our knowledge, there is no experimental research that does this either. Clearly, little is known about the interaction of these two effects. We aim to fill this gap by using a series of laboratory experiments to investigate the effects of strategic timing decisions on coordination under asymmetric information—i.e., when strategic complementarities and information externalities coexist.

The design of our experiment is based on the global coordination game introduced by Carlsson and van Damme [17]. Since the main focus of the model is on strategic externalities and asymmetric information about the fundamentals of the economy, it provides us with the ideal framework. Incorporating endogenous timing into this model allows us to analyze the two effects of timing, which we discussed above. Our theoretical exercise aims to build insights and develop hypotheses by analyzing a model that is more general than the experimental treatment. We first characterize the equilibrium of a simple two-player global coordination game that allows endogenous timing. Our analysis shows that the complementarity and the learning effects of timing allow players to internalize the returns from coordination, and that in equilibrium, strategic delay becomes a coordination device. In particular, we find that a player who has optimistic beliefs invests earlier, and a player who has more pessimistic beliefs invests later, if at all.

The reason behind the optimistic investor's behavior is twofold. As a direct effect, optimistic expectations of higher returns makes investment attractive. As an indirect effect, an optimistic player realizes that his investment will make the other player more optimistic, and the other player will then invest in the following periods if he has not already done so. This, in turn, further increases the optimistic investor's expectation of higher returns, which is based on strategic externalities. The equilibrium behavior of the pessimistic investor is as follows: because of his pessimistic beliefs, he not only expects a low return, but he also believes that the other investor holds pessimistic beliefs as well. Therefore, he finds it optimal to wait. If he observes an investment in the following periods he becomes less pessimistic and adjusts his decision accordingly.

This characterization allows us to study the effect of endogenous timing on welfare. To this end, we compare equilibrium welfare in global coordination games with endogenous timing to equilibrium welfare in global coordination games without endogenous timing, which we refer to as sequential and simultaneous games, respectively. First, as a benchmark, we eliminate incomplete information to focus on the complementarity effect of timing. It is well known that there are multiple equilibria in the simultaneous game. We take the risk-dominant equilibrium as the benchmark since it is also the limit equilibrium of the simultaneous game as the player's signal becomes fully informative. In the complete information sequential game, the subgame perfect equilibrium is the one where both players invest in the first period if and only if the return of the investment is positive. When we compare the two games, we see that the equilibrium of the sequential game is always efficient, whereas the equilibrium of the simultaneous game is efficient only some of the time. That is, the complementarily effect of timing by itself can entirely eliminate all the inefficiency present in the simultaneous game.

While we are able to make sharp comparisons when there is complete information, we need to perform a computational analysis to make comparisons when the signals are not fully informative. When the state is high enough we find that sequential game outperforms the simultaneous game and conversely that the simultaneous game outperforms the sequential game when the state is low. This finding is quite intuitive because the risk-dominant equilibrium is averse to the risk of coordination failure, which is more pronounced when the state is low.

This analysis provides a more interesting result when we consider the difference in ex ante (equilibrium) welfare between the sequential and simultaneous games as a function of the informativeness of the signals. We find that the difference of ex ante welfare increases as the informativeness of signals decreases. This suggests that it is especially when the signals are less informative that endogenous timing leads to an improvement in welfare.

The laboratory experiment provides us with an environment in which we can observe empirical regularities while we control for asymmetric information—a variable that is hard to observe or identify (and, hence, control for) in market data. We base the experiment on a simple, special case of the model and vary the informativeness of private information in both simultaneous and sequential games to obtain a rich data set. A simple analysis of the data shows that the subjects' actual behavior is consistent with theoretical equilibrium behavior. For example, although there are three periods in which the subjects can make their decision, there are negligibly few subjects who delay investment to the third period. So if a subject has optimistic beliefs, he invests in the first period; if his beliefs are more pessimistic, however, he waits until the second period to invest. We also see that a subject who delays his decision invests in the second period only if he observes the other subject make an investment and if he is not too pessimistic. Otherwise he never invests.

Our first result indicates that in the simultaneous game subjects coordinate more than is predicted by the theory; that is, subjects overinvest. This result is consistent with the findings of Heinemann et al [37] and Duffy and Ochs [21]. In the sequential game, in contrast, the behavior of the subjects who invest in the first period is quite consistent with the theory. However, the behavior in the second period deviates from the equilibrium predictions. In fact, we show that in the sequential game, while there are subjects whose behavior can be explained by the theory, there are also subjects who delay investment, even though (according to theory) they should have invested in the first period. We call this type of behavior "confidence seeking." These latter subjects are those who prefer to wait to obtain more information before deciding whether to invest (i.e., to see what investment decisions the other subject makes), even though immediate investment is justified, based on their private information (i.e., the information they already have).

Another main finding of our experiment is that endogenous timing is a significant instrument for enhancing welfare, except when there is no uncertainty. When the subjects have complete information, the coordination rates between the simultaneous and sequential games are indistinguishable. This suggests that the complementarity effect of timing is not a strong determinant of the outcome. When there is uncertainty, we find that coordination about the efficient outcome is uniformly higher in the sequential game than in the simultaneous game. Consistent with our theoretical findings, the gap between the coordination rates of simultaneous and sequential cases increases when private information becomes less informative. That is, the more asymmetry of information there is between the players, the more endogenous timing facilitates coordination. We confirm the same results when making similar comparisons in terms of subjects' actual earnings. Overall, we conclude that in the lab the learning effect of timing is stronger than the complementarity effect of timing.

The rest of the paper is organized as follows: Sections 2 and 3 provide the positive and normative analysis of the model. We pose the empirical questions in Section 4. Section

5 provides an analysis of the data and seeks answers to the questions. We conclude in Section 6. All the proofs are relegated to an Appendix.

1.1 Related Literature

1.1.1 Theoretical Literature

Since our experimental design is based on Carlsson and van Damme [17], for a thorough review of global games literature we will refer the reader to Morris and Shin [47], Morris [45], and the references therein. The global games literature takes the view that the complete information assumption accounts for the multiplicity of equilibria inherent in coordination games. A key result in this literature is that multiplicity vanishes once the economy is perturbed by small failures of common knowledge. The goal of the paper is different since we focus on questions that are concerned with the effects of endogenous timing on coordination under asymmetric information. In other words, asymmetric information is a defining feature of the environment rather than a vehicle to approximate the complete information model.

Dasgupta [19] is the closest paper to our work.[1] It provides an analysis of a two-period game with endogenous timing and a continuum of players. In the second period of the game, players who still have an option to invest observe an additional private signal that provides information about the aggregate level of investment in the first period. The second period signal in Dasgupta [19] differs from the information conveyed in our model by an investment decision in two respects: it is privately observed and it is subject to idiosyncratic noise. This modeling choice has implications in terms of the interplay between information and timing, which is the main focus of the present paper. In Dasgupta [19], players who invest in the first period know that they collectively play a role in determining second-period signals. Therefore, they take into account the aggregate influence on investment decisions in the second period, yet, because of the continuum of players, the individual signaling effect is negligible. Under specific distributional assumptions, Dasgupta [19] shows that if private information is sufficiently precise, the game has a unique equilibrium in monotone strategies. However, because the signaling effect is negligible the equilibrium does not achieve efficiency at the complete information limit.

Our paper also shares some common elements with the literature on observational

---

[1] Angeletos and Werning [4], Angeletos et al. [1] and Heidhues and Melissas [36] also share some similarities. Among other results, the first paper shows that the endogenous transmission of information can lead to a multiplicity of equilibria, while the second shows, in a dynamic model of regime change that the interplay of endogenous and exogenous learning about the economy also generates a multiplicity of equilibria. Finally, Heidhues and Melissas [36] examine the role of cohort effects and characterize the conditions under which uniqueness of equilibrium is established.

learning in environments with pure information externalities. A few papers focus on endogenous timing and hence strategic delay (cf. Caplin and Leahy [13], Chamley and Gale [11], Gul and Lundholm [34] and Levin and Peck [42]). Under pure information externalities, an investor can wait and observe the behavior of other investors to gain additional information on the uncertain returns of investment. Investment is undertaken if the current expected returns are higher than the value of future information gains net the exogenous cost of delay. In our paper, the additional presence of payoff externalities causes a fundamental change since now an investor must weigh the benefits of moving early to transmit information to others against the benefits of waiting to observe other investors' decisions.

Finally there is also a large literature that studies the role of timing in coordination games. Although many papers, such as Farrell and Saloner [25, 26], Katz and Shapiro [40, 41] Farrell [24], Bolton and Farrell [7], Gale [31], Choi [10], are similar in terms of the structure of the problem, they are fundamentally different in the questions they study.

1.1.2 Experimental Literature

Like the theoretical literature on global games, the experimental literature can be categorized as dealing with either static or dynamic global games. Heinemann et al. [37], which tests Morris and Shin [46], is the pioneering experimental work on static global games. This paper presents three main findings: First, differences in observed behavior under complete and incomplete information are not significant.[2] Second, behavior is consistent with the comparative statics of global games. Third, subjects cooperate more often than the predictions of the theory.

Duffy and Ochs [21] is the bridge between static and dynamic global games experiments. In a similar environment to the complete information treatments of Heinemann et al. [37], they study behavior in both static and dynamic games.[3] In contrast, to their focus on complete information, our experiments examine behavior in both static and dynamic games under incomplete information, allowing us to provide a clear test of the theory of dynamic global games that we consider.

In its static treatments, Duffy and Ochs [21] elicits subjects' strategies and finds that actual decisions are often consistent with the implied, elicited cutoff strategies, though they also find that there is substantial variance in individual cutoffs and that cutoffs vary with changes in the environment in ways not predicted by the theory. Comparing the

---

[2] Similarly, Cabrales et al. [12] tests Carlsson and van Damme [17] and provide further support that complete and incomplete information environments lead to similar behavioral patterns. They show that behavior converges to theoretical predictions as the subjects become more experienced.

[3] Fehr and Shurchkov [28] and Costain et al. [18] also conduct dynamic global games experiments, though because the underlying models that they study have multiple equilibria, they are largely separate from our focus.

dynamic and static games, Duffy and Ochs [21] finds that cutoffs do not differ in the two treatments. That is, while subjects have lower cutoffs than theory predicts in the static game, they have higher cutoffs than the efficient subgame perfect equilibrium threshold in the dynamic games. Moreover, in the dynamic games, they find that subjects adopt a "wait-and-see" approach, which often leads to substantial delay.

Our experimental data share some similarities with these papers. Like Heinemann et al. [37] and Duffy and Ochs [21], we find that subjects have lower thresholds than predicted by the global games theory in our static environment. Also, like Duffy and Ochs [21], we find that subjects in our dynamic treatments have higher thresholds than predicted by our theory, and that the thresholds in the dynamic and static treatments are indistinguishable under complete information. Since, unlike Duffy and Ochs [21], our experiment involves incomplete information, we find that behavior is still distinguishable (in particular, thresholds are lower and the frequency of coordination is higher in the dynamic games than in the static games) and report additional results concerning existence of incomplete information.

2 Theory

2.1 Preliminaries

Consider the following game à la Carlsson and van Damme [17]. Two players, $i = 1, 2$, make a binary investment decision. The set of actions is $A := \{\text{I}, \text{W}\}$; we interpret action I as *investing* and W as *waiting*. While W is a safe action, whose payoff is normalized to zero, the return of investment is determined by a random variable Θ. Given a realization θ, the returns at each action profile are determined as in Figure 1.

Figure 1: Player's Returns for a Realization of Θ

	I	W
I	θ , θ	$\theta - 1$, 0
W	0 , $\theta - 1$	0 , 0

The action I exhibit strategic complementarities. If a player invests, the return of investment is higher when the other player also invests.

Each player's prior belief about Θ is represented by a measure over the real line. The true state, θ, is realized prior to investment decisions. Following the realization of θ, each player receives a private signal. Player i's signal is determined by the random variable X_i

defined as

$$\mathsf{X}_i := \Theta + \mathsf{E}_i,$$

where E_i is a real valued random variable. We assume that E_1 and E_2 are identically and independently distributed, and that E_i and Θ are independent for $i = 1, 2$. Without loss of generality, we also assume that $\mathbf{E}[\mathsf{E}_i]$=0.[4]

All the random variables are defined on the Borel probability space. They are absolutely continuous with respect to Lebesque measure, hence the probability densities are well defined. For a generic random variable Y, F_{Y} and f_{Y} denote the distribution and density functions respectively.

We assume that the players are Bayesian. So, when player i receives a signal x_i he updates his belief about Θ by Bayes' rule. Player i's posterior belief about Θ, conditional on observing x_i, is represented by the distribution function $F_{\Theta}(\cdot|x_i)$. A player also forms beliefs about the other player's signal. Player i, conditional on observing x_i, updates his belief about the signal of the other player, which we denote by $F_{\mathsf{X}_j}(\cdot|x_i)$.

Finally we assume that $f_{\mathsf{X}_i}(\cdot|\theta)$ satisfies the monotone likelihood ratio property (MLRP) for each $i = 1, 2$:

$$\frac{f_{\mathsf{X}_i}(x|\theta)}{f_{\mathsf{X}_i}(x|\theta')} \geq \frac{f_{\mathsf{X}_i}(x'|\theta)}{f_{\mathsf{X}_i}(x'|\theta')}$$

for $x \geq x'$ and $\theta \geq \theta'$. The MLRP is a standard assumption that provides a monotone environment in the study of many economic problems. In the current context, it means that if the return of investment is high it is more likely to receive a higher signal. This condition has the following implications that we rely upon throughout the analysis.

LEMMA 1. *If $f_{\mathsf{X}_i}(\cdot|\theta)$ satisfies* MLRP *for each $i = 1, 2$, the following statements hold.*
(i) $f_{\Theta}(\cdot|x_i)$ satisfies MLRP.
(ii) $f_{\mathsf{X}_j}(\cdot|x_i)$ satisfies MLRP.

The first result states that under MLRP, a high signal means that it is more likely that the realization of Θ is high. The second result shows that a similar monotonicity holds for the signal of the other player. That is, if a player receives a high signal, it is more likely that the opponent receives a high signal.

Given Lemma 1, the following results show that the monotonicity provided by the MLRP translates into monotonicity in the expectation of Θ.

LEMMA 2. *If $f_{\mathsf{X}_i}(\cdot|\theta)$ satisfies* MLRP *for each $i = 1, 2$, the following statements hold.*
(i) $\mathbf{E}[\Theta|x_i, x_j \geq k] \geq \mathbf{E}[\Theta|x_i]$ for any $k \in \mathbb{R}$.
(ii) $\mathbf{E}[\Theta|x_i, x_j \leq k] \leq \mathbf{E}[\Theta|x_i]$ for any $k \in \mathbb{R}$.
(iii) $\mathbf{E}[\Theta|x_i, x_j \geq k]$ and $\mathbf{E}[\Theta|x_i, x_j \leq k]$ are increasing in k.

---

[4] Throughout the paper $\mathbf{E}[\,\cdot\,]$ denotes the expectation operator.

Throughout the paper we assume that there exist signals $\underline{x}$ and $\bar{x}$ such that $\mathbf{E}[\Theta|\underline{x}] = 0$ and $\mathbf{E}[\Theta|\bar{x}] = 1$. Therefore W is a dominant strategy for $x_i < \underline{x}$ and I is a dominant strategy for $x_i > \bar{x}$.

2.2 The Game

The game consists of τ periods: $t = 1, 2, \cdots, \tau$. In each period, each player i is supposed to take an action $a_i^t \in \{\text{I}, \text{W}\}$. The action I is *irreversible*, but W is *reversible*. Therefore, if $a_i^t = \text{I}$, then $a_i^{t+1} = \text{I}$, while if $a_i^t = \text{W}$, player i can still choose between actions I and W in the subsequent periods. However, action I is costly in periods $t > 1$: If a player switches from action W to I in period t then he pays a cost of $(t-1)c \geq 0$.[5]

Denote the profile of actions taken in period t by $a^t = (a_1^t, a_2^t)$ and the initial history by $\varnothing$. $\mathcal{H}$ is the set of all histories. We assume that players have perfect information about the history, i.e. in period $t > 1$ a player can observe all the actions taken in the previous periods. The players receive a private signal determined by X_i before the game starts. The assumptions on the information structure are as we discussed before. Throughout the paper the solution concept is the *perfect Bayesian equilibrium*.

For a given θ, players' payoffs are determined by the sequence of actions taken in τ periods:

$$u_i\big((a^t)_{t=1}^{\tau}\big) = \begin{cases} 0 & \text{if } a_i^{\tau} = \text{W}, \\ \theta - (t-1)c - \mathbb{1}_{a_j^{\tau}=\text{W}} & \text{if } a_i^{t-1} = \text{W and, } a_i^t = \text{I}, \end{cases}$$

where $\mathbb{1}_{a_j^{\tau}=\text{W}}$ is an indicator function that takes value 1 if $a_j^{\tau} = \text{W}$ and 0 otherwise.

A strategy of a player is a measurable function that maps each history and signal to an action. In our equilibrium analysis, we will focus on *monotone* strategies, which we define next.

Definition 1. *A strategy is a measurable map $\sigma : \mathbb{R} \times \mathcal{H} \mapsto \{\text{I}, \text{W}\}$. We say that player i's strategy is monotone if for each $h \in \mathcal{H}$ there exists a $k_i^h \in \overline{\mathbb{R}}$ such that $\sigma_i(x_i, h) = \text{I}$ if and only if $x_i \geq k_i^h$.*[6]

A monotone strategy simply states that if a player invests for a signal x_i, then he also invests for signals greater than x_i. Thus, a monotone strategy is characterized by thresholds, one for each possible history. Under a monotone strategy, a player i, who observes history h, invests in period t if $x_i \geq k_i^h$; otherwise, he waits.

---

[5] All the qualitative aspects of our results remain the same for any cost function that is increasing in time.

[6] $\overline{\mathbb{R}}$ denotes the extended real line, i.e., $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, \infty\}$. For instance, if $k_i^h = \infty$, then we say that "player i never invests after having observed history h."

2.2.1 SIMULTANEOUS GAME ($\tau = 1$)

If $\tau = 1$, the game reduces to a simultaneous game (henceforth SIM), which is analyzed extensively in the literature of global games. For completeness, we will briefly review the key aspects of the game.

Since there is only one period a single threshold characterizes the players' monotone strategies. So, we will write $\kappa_i := k_i^{\varnothing}$ to denote player i's threshold. Note that, if player j follows a monotone strategy with threshold κ_j, then player i expects him to invest with probability $1 - F_{\mathsf{X}_j}(\kappa_j|x_i)$, and wait with probability $F_{\mathsf{X}_j}(\kappa_j|x_i)$. Hence, player i's expected payoff from investment is $\eta(x_i, \kappa_j) := \mathbf{E}[\Theta|x_i] - F_{\mathsf{X}_j}(\kappa_j|x_i)$. Consequently, player i's best response is to invest if and only if $\eta(x_i, \kappa_j) \geq 0$. If we restrict our attention to symmetric equilibria in monotone strategies, an equilibrium with threshold κ must satisfy the indifference condition $\eta(\kappa, \kappa) = 0$

The existence of equilibrium is straightforward, yet uniqueness of equilibrium depends on the behavior of $F_{\mathsf{X}_j}(\kappa|x_i = \kappa)$.[7] The uniqueness of the equilibrium is discussed in the literature (viz. Frankel et al. [30], Morris and Shin [47]). For instance, Morris and Shin [47] shows that when Θ and X_j are normally distributed with variances σ^2 and σ_ϵ^2, there exists a unique equilibrium if the ratio $\sigma_\epsilon^2/\sigma^2$ is small enough. In other words, if players' private information is sufficiently informative compared to the common prior, then the uniqueness of equilibrium is established.

2.2.2 SEQUENTIAL GAME ($\tau \geq 2$)

Players' strategic considerations change dramatically when the game becomes sequential (henceforth SEQ). In the SEQ game, on one hand a player can wait in order to benefit from the observations he can make in later periods; on the other hand, he can invest in the current period, avoid the cost of investment in the future, and more importantly, reveal information to the other player. This information may convince the other player to invest and they can both benefit from the payoff externality.

We begin our analysis with the preliminary observation that in a monotone-strategy equilibrium, once both players wait, there is no investment in the future.

LEMMA 3. *Let $\sigma = (\sigma_1, \sigma_2)$ be an equilibrium of the* SEQ *game in monotone strategies. If $a^t = (\mathrm{W}, \mathrm{W})$ then $a^{t+1} = (\mathrm{W}, \mathrm{W})$ for all $t < \tau$.*

The intuition behind this result is simple. At any point in time a player decides whether to invest or not by comparing the expected payoff of investing, and the option value of

---

[7] In fact, by the iterated elimination of strictly dominated strategies, when there is a unique equilibrium in monotone strategies, then there are no other equilibria, monotone or otherwise.

waiting. In an equilibrium with monotone strategies, observing that the other player's action is W reveals *bad* news, which makes the player more pessimistic and hence lowers the expected payoff of investing. As a result, a player who has not already invested, finds it even less profitable to invest after observing W.

Lemma 3 simplifies our equilibrium analysis to a great extent since it implies that in SEQ investment takes place in the first two periods. This is so simply because time is costly and players take an investment decision as soon as there is no option value of delay. This leads to the following equilibrium characterization.

PROPOSITION 1. *There exists a $c^* \in \mathbb{R}_+$ such that if $c < c^*$ there exists a unique equilibrium σ of the* SEQ *game in monotone strategies. The equilibrium is symmetric and characterized by*

$$\sigma_i(x_i, h) := \begin{cases} \text{I} & \text{if } h = \varnothing \text{ and } x_i \geq k^1, \\ \text{I} & \text{if } a_j^1 = \text{I} \text{ and } x_i \geq k^2, \\ \text{W} & \text{otherwise,} \end{cases}$$

such that

$$\begin{aligned} \mathbf{E}[\Theta | x_i = k^2, x_j \geq k^1] &= c, & (1) \\ \mathbf{E}\left[\Theta | x_i = k^1, x_j < k^1\right] - F_{X_j}(k^2 | x_i = k^1, x_j < k^1) &= -c \frac{1 - F_{X_j}(k^1 | x_i = k^1)}{F_{X_j}(k^1 | x_i = k^1)}, & (2) \end{aligned}$$

for $i, j = 1, 2$.

If $c \geq c^*$, there exists a symmetric equilibria σ of the SEQ *game. The equilibrium strategies are monotone, symmetric and characterized by*

$$\sigma_i(x_i, h) := \begin{cases} \text{I} & \text{if } h = \varnothing \text{ and } x_i \geq k, \\ \text{W} & \text{otherwise,} \end{cases} \quad \text{such that } \mathbf{E}[\Theta | x_i = k] - F_{X_j}(k | x_i = k) = 0$$

for $i, j = 1, 2$.

Proposition 1 fully characterizes the equilibrium of the SEQ game. There are two types of equilibria depending on the level of cost. c^* is the level of cost above which even a player who waits in the first period and observes an investment would not invest in the next period regardless of his signal. In other words, for any $c \geq c^*$ the game is effectively simultaneous.

When the cost is small enough, the unique equilibrium of the game is characterized by two thresholds $k^1 \geq k^2$. A player invests in period one for signals above k^1; otherwise, he waits. If his signal is between k^2 and k^1, he invests in period two only when he observes an investment, otherwise he waits. If his signal is less than k^2, then he never invests.

The condition (1) in Proposition 1 determines the optimal threshold (k^2) of a player who did not invest in the first period but observed the other player investing. The observation that the other player invested in the first period reveals that his signal is above k^1. Having updated his beliefs accordingly, the player invests in the second period for any signal above k^2 since the expected value of investing exceeds the cost.

The condition (2) determines the optimal threshold in the first period for a given k^2. Basically it compares the ex ante expected value of waiting with the expected value of investing today. In both cases a player gets a payoff of θ when the other player invests, whereas in the latter case he gets $\theta - 1$ when the other player does not invest in the first period. Hence, the equality is conditioned on the event that the other player does not invest in the first period (i.e. $x_j < k^1$), and determines the optimal threshold k^1 that balances the trade off between waiting and investing in the first period.

When the cost is high enough the game becomes a simultaneous game since the option value of delay never exceeds the cost of investment. Therefore, in this case the equilibrium is determined as we discussed in Section 2.2.1.

3 Discussion of Equilibria

The equilibrium analysis of the seq and sim games sheds light on the *learning* and *complementarity* effects of timing. Let us first discuss the benchmark cases of sim and seq under complete information. Throughout our discussion let us assume that $\theta \in [c, 1]$, so that investing in the second period remains a feasible option.

The equilibria of seq and sim under complete information are well-known. While there are multiple equilibria in the sim game, seq game has a unique subgame perfect equilibrium in which both players invest in the first period. In the sim game, the reason behind multiple equilibria is the self-fulfiling beliefs: if both players believe that the other player plays I (W) then it is optimal to play I (W). On the other hand, the uniqueness of the subgame perfect equilibrium in the seq game depends on players' reasoning that if they invest in the first period then the other player—even if he has not already invested—will invest in the second period for any $\theta \geq c$. So, under complete information, the possibility that the players can choose the timing of their decisions has an important impact on the equilibrium prediction of the game, through the complementarity effect of timing.

When there is uncertainty about the fundamentals, the equilibrium prediction changes dramatically in the sim game under asymmetric information. In the Bayesian equilibrium of the game, when both players have a signal above the equilibrium threshold $\kappa > 0$, they coordinate on the outcome (I,I). Similarly, they coordinate on the outcome (W,W) when both signals are below κ. Finally, the players fail to coordinate when one signal is above and the other one is below κ. The literature on global games shows that these failures vanish as the players' signals become infinitely informative. In fact, in the limit there exists a unique equilibrium. We state this formally in the following proposition.[8]

PROPOSITION 2. *Let* $\{\mathsf{E}_i^n\}$ *and* $\{\mathsf{E}_j^n\}$ *be sequences of independent random variables that weakly converge to the Dirac measure* δ_0.[9] *Define* $\mathsf{X}_i^n := \Theta + \mathsf{E}_i^n$ *and* $\mathsf{X}_j^n := \Theta + \mathsf{E}_j^n$. *For any* n, *there exists an equilibrium, which can be characterized by a* $\kappa^n \in (0,1)$. *There exists a unique* $\kappa \in (0,1)$ *such that any sequence of equilibria* $\{\kappa^n\}$ *converges to* κ.

An immediate consequence of Proposition 2 is that even though the efficient outcome is to invest when the expectation of θ is positive, this does not necessarily occur in the limit equilibrium. More generally, Carlsson and van Damme [17] shows that in the limiting equilibrium players coordinate on the risk-dominant equilibrium à la Harsanyi and Selten [35]. Risk dominance is an equilibrium refinement that selects the equilibrium that is the least risky in terms of players' subjective uncertainty about the strategy of the other player. While this explanation is based on the ad hoc notion of subjective uncertainty about players' strategies, the approach of Carlsson and van Damme [17] provides a robust intuition. Carlsson and van Damme [17] notes that in the global game approach "[...] uncertainty about your opponent's choice of actions appears when your observation is close to the point where he switches from one action to the other. Hence, the optimal choice of switching point requires the players to consider the relative riskiness of the two equilibria."

In the case of SEQ, under complete information, there is a unique subgame perfect equilibrium because a player *knows* that his investment in the first period *guarantees* coordination. Under asymmetric information, the explanation behind the perfect Bayesian equilibrium is similar. Precisely, if a player moves first, the probability that the other player will invest in the second period increases (though it does not guarantee as in the complete information case). The investment in the first period affects the probability of coordination in two ways: First, as in the complete information case, there is no risk of miscoordination for a player who did not invest in the first period. Second, since a player invests in the first period only if his signal is high enough (above k^1), observing investment makes the beliefs of the other player more optimistic about the overall profitability of investment. As a result, the equilibrium exhibits *sorting*: the players whose signals are high enough move first, while those who do not invest in the first period benefit from the information

---

[8] Note that the characterization of equilibrium in the following statement is provided in the Proposition 1 for the case $c \geq c^*$.

[9] For $E \subset \mathbb{R}$, a Dirac measure δ_0 satisfies $\delta_0(E) = 1$ if $0 \in E$, and $\delta_0(E) = 0$ otherwise. Let F^n and F be the distribution functions of the random variables E^n and E, respectively. A sequence $\{E^n\}$ of random variables is said to converge weakly to a random variable E if $\lim_{n\to\infty} F^n(x) = F(x)$, for every number $x \in \mathbb{R}$ at which F is continuous.

revealed from the investment decision. The sequential structure of SEQ makes it possible to exchange information (learning effect) and use timing as a coordination device (complementarity effect).

Morris [45] takes the view that "[s]ince complete information, or common knowledge of payoffs, is surely always an idealization anyway, the play selected in the global game with small noise can be seen as a prediction for play in the underlying complete information game." As we discussed earlier, this approach leads to a unique equilibrium as the signals become infinitely informative in the simultaneous coordination games. Hence, this particular perturbation is seen as a refinement used to select a unique equilibrium. The same is not true in the SEQ game since the SEQ game under complete information already has a unique subgame perfect equilibrium. More importantly, our purpose in introducing asymmetric information and endogenous timing is to understand the complementarity and learning effects of timing. Nevertheless, for completeness, we show that in the limit case of SEQ when the private signals become increasingly informative, the unique limiting equilibrium is the subgame perfect equilibrium of the SEQ game under complete information. In other words, there is no discontinuity in the limit of the SEQ game.

PROPOSITION 3. *Let* $\{\mathsf{E}_i^n\}$ *and* $\{\mathsf{E}_j^n\}$ *be sequences of independent random variables that weakly converge to the Dirac measure* δ_0. *Define* $\mathsf{X}_i^n := \Theta + \mathsf{E}_i^n$ *and* $\mathsf{X}_j^n := \Theta + \mathsf{E}_j^n$. *Then* $\lim_{n\to\infty}(k_1^n, k_2^n) = (c, c)$.

3.1 WELFARE

Propositions 2 and 3 suggest that while endogenous timing leads to the efficient outcome in the limit, the same is not necessarily true in the SIM game. In order to see this let us define the value of both games in the equilibrium for a given information structure, conditional on the true state θ. Note that since we can possibly have multiple equilibria in SIM, the value depends on the choice of equilibrium (κ). The value of SIM for a player i is:

$$v_i(\theta,\kappa) := \big(1 - F_{\mathsf{X}_i}(\kappa|\theta)\big)\big(\theta - F_{\mathsf{X}_j}(\kappa|\theta)\big).$$

That is, conditional on θ, a player receives a signal above the threshold κ with probability $\big(1 - F_{\mathsf{X}_i}(\kappa|\theta)\big)$. In this case, he invests and receives an expected payoff of $\big(\theta - F_{\mathsf{X}_j}(\kappa|\theta)\big)$. Otherwise, he does not invests and hence gets 0.

Similarly, we define the value of SEQ for a player i as

$$w_i(\theta) := \big(1 - F_{\mathsf{X}_i}(k^1|\theta)\big)\big(\theta - F_{\mathsf{X}_j}(k^2|\theta)\big) + \big(F_{\mathsf{X}_i}(k^1|\theta) - F_{\mathsf{X}_i}(k^2|\theta)\big)\big(1 - F_{\mathsf{X}_j}(k^1|\theta)\big)(\theta - c). \quad (3)$$

Conditional on θ, a player receives a signal above k^1 with probability $\left(1 - F_{\mathsf{X}_i}(k^1|\theta)\right)$ and invests with an expected value of $\left(\theta - F_{\mathsf{X}_j}(k^2|\theta)\right)$. If his signal is between the thresholds k^1 and k^2—with probability $\left(F_{\mathsf{X}_i}(k^1|\theta) - F_{\mathsf{X}_i}(k^2|\theta)\right)$—then he only invests if the other player invests in the first period, which happens with probability $\left(1 - F_{\mathsf{X}_j}(k^1|\theta)\right)$, and he then gets $\theta - c$. In all other cases he gets zero.

As the benchmark case we can now compare the two games in the limiting case where the signals become infinitely informative, i.e., the players almost surely know the state of the world. We define $\mathsf{X}_i^n := \Theta + \mathsf{E}_i^n$ for $i = 1, 2$ as before and define $v_i^n(\theta, \kappa)$ and $w_i^n(\theta)$ for $\mathsf{X}_i^n, \mathsf{X}_j^n$ correspondingly. As a corollary to Propositions 2 and 3 we can make the following statement.

COROLLARY 1. *Let* $\{\mathsf{E}_i^n\}$ *and* $\{\mathsf{E}_j^n\}$ *be sequences of independent random variables that weakly converge to the Dirac measure* δ_0. *Let* $c >$ *be small enough. For any sequence of equilibria* $\{\kappa^n\}$, $\lim_{n\to\infty} v_i^n(\theta) = \theta \mathbb{1}_{\{\theta:\theta>\kappa\}}$ *and* $\lim_{n\to\infty} w_i^n(\theta) = \theta \mathbb{1}_{\{\theta:\theta>c\}}$, *where* $\mathbb{1}_E$ *is an indicator function that takes value 1 when event* E *occurs and value 0 otherwise.*

Corollary 1 provides a sharp welfare comparison in the complete information limit yet it does not provide a general result for any level of uncertainty. Although we are not able to provide an analytical result regarding a general comparison, in order to get a sense of welfare in the presence of noise in both the SEQ and the SIM games, Figure 2 plots the computations of the welfare difference of the two games under two different information structures. The figure on the left assumes that Θ is uniformly distributed between 20 and 50, and for a realization of θ, signals are uniformly distributed between $\theta - e$ and $\theta + e$, where $e > 0$ is the index that determines the informativeness of the signals.[10] Let us define $\varphi(\theta; e) := w_i(\theta; e) - v_i(\theta; e)$ to be the welfare difference between SEQ and SIM as a function of θ when the parameter is e. The right-hand figure depicts the case in which the prior on θ is diffuse and for a given state θ, signals are normally distributed with mean θ and variance σ^2. Similarly, in this case we will write $\varphi(\theta; \sigma^2) := w_i(\theta; \sigma^2) - v_i(\theta; \sigma^2)$ to be the welfare difference between SEQ and SIM as a function of θ when the variance of the signal distribution is σ^2.[11]

In both examples we see that when θ is low we get $\varphi(\theta; \cdot) < 0$, indicating that welfare is actually higher in the SIM game. This follows because in the SIM game, since the threshold κ is higher than k^1, it is unlikely that a player will receive a signal above his threshold, causing him to invest erroneously. In contrast, in the SEQ game, since the period 1 threshold is lower, it is more likely that a player will receive a signal above the threshold and,

---

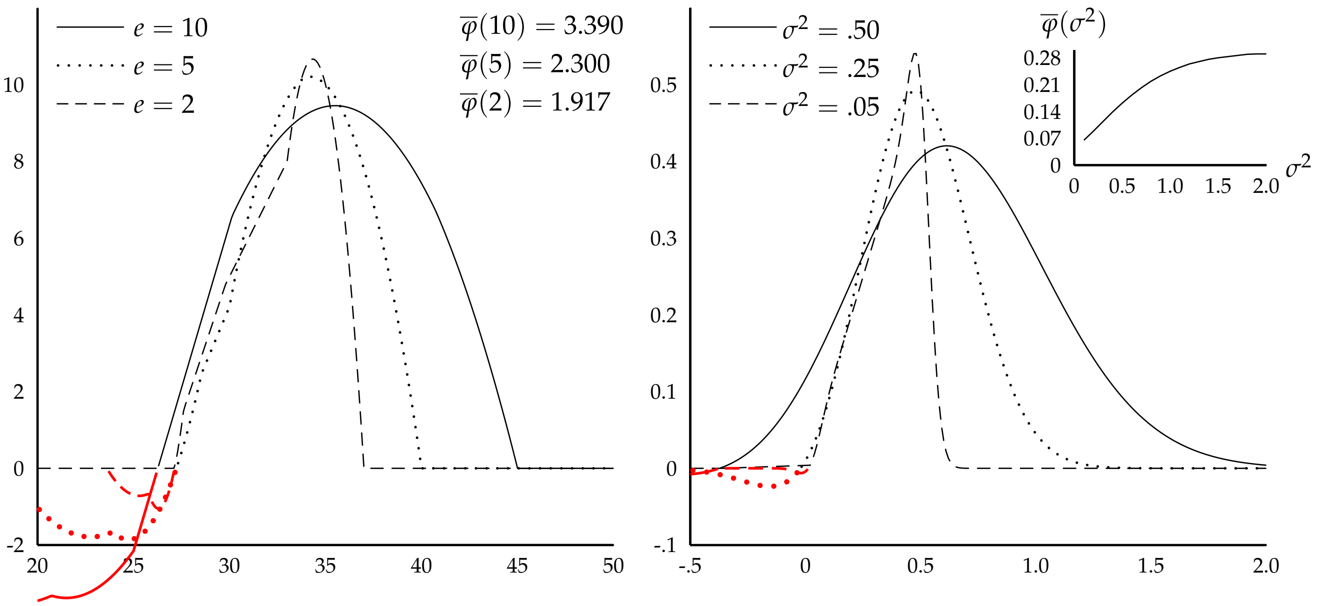
[10] These are the same parameters that we use in the design of the experiment.

[11] In both cases there exists a unique equilibrium of the SIM game, hence we drop κ in $v_i(\theta, \kappa; e)$ and $v_i(\theta, \kappa; \sigma^2)$.

erroneously invest. As θ increases, $\varphi(\theta; \cdot)$ becomes positive, hence for intermediate values of θ welfare is higher in seq than in sim. Eventually, $\varphi(\theta; \cdot)$ starts to decline since for sufficiently high values of θ, both players in the sim game are very likely to receive signals above the threshold. As θ becomes large enough the welfare difference between the two games vanishes.

Note that although there is no clear comparative static for $\varphi(\theta; \beta)$ as $\beta = e, \sigma^2$ changes, it seems $\overline{\varphi}(\beta) := \int \varphi(\theta; \beta) dF(\theta)$ is increasing in $\beta = e, \sigma^2$. That is, *on average* the welfare difference between seq and sim is increasing as the informativeness of the signals decreases. This is reported on the upper right corner of both figures; on the left we see that $\overline{\varphi}(10) > \overline{\varphi}(5) > \overline{\varphi}(2)$ and on the right $\overline{\varphi}(\sigma^2)$ is monotonically increasing in σ^2.

Figure 2: Welfare Comparison of seq vs. sim


Uniform Prior & Uniform Noise

Diffuse Prior & Normal Noise

4 Empirical Questions

The theoretical predictions provide us with a number of empirical questions that we can pose in an experiment, which implements a simplified version of the model. In this section we discuss and develop our main hypotheses.

Thresholds. The equilibrium analysis is based on the assumption that players use monotone strategies. To be precise, while in sim the unique equilibrium is in monotone strategies, in SEQ we assume that the strategies are monotone and we characterize the equilibrium. This leads to the obvious question of whether subjects' behavior is consistent with the use of monotone strategies. This is important for at least two reasons. First, it can help us to understand whether the equilibrium that we characterized provide a meaningful benchmark to analyze the data. Second, we can analyze the data under the light of theoretical comparative statics, and if there are systemic deviations, we can question when and how subjects' behavior cause these deviations.

RATIONALITY AND VALUE OF INFORMATION. Recall that in the equilibrium of the SEQ game we can observe investment only in the first two periods. This is because time is costly and because investing reveals good news, while not investing reveals bad news. By allowing a longer time horizon in the experiment, we can test whether subjects' behavior is consistent with this result. Moreover, assuming that subjects' behavior is consistent with threshold strategies, we can further ask how well the subjects understand the value of information. As was the case above, this is important since it makes us confident that our analysis of the data to understand the role of timing can be made using the theory as a benchmark.

TIMING AND COORDINATION. Under complete information, an important distinction between SIM and SEQ games is that the latter has a sharp prediction while the former predicts multiple equilibria. Moreover the reasoning of the unique equilibrium is that players invest in the first period because they know that the other player will surely invest once they observe investment. This argument provides us with a very clean test to understand the role of timing as a coordination device. In other words, in the absence of any information asymmetries, we can test whether time works as a coordination device by comparing SEQ and SIM.

In the asymmetric information case, timing has an additional role; namely an early investor reveals good news to his opponent about the profitability of investment. That leads to the question of how well subjects can use timing to convey information to the other player. We can also investigate how the dynamic nature of the game lead to improved coordination, relative to the static game, in the existence of asymmetric information?

WELFARE. Clearly, the role of timing has important implications in terms of welfare. We have a sharp limit result that compares welfare in SIM and SEQ, which can be tested empirically. Although we do not have analytic results that compare welfare in the presence of asymmetric information, our simulations indicate that the difference of average welfare between SEQ and SIM increases as the asymmetry of information increases. That is to say, endogenous timing is *more* welfare enhancing when the environment is more noisy.

5 EXPERIMENT

5.1 EXPERIMENTAL DESIGN

The experiment was run at the Experimental Economics Laboratory of the Center for Experimental Social Sciences (C.E.S.S.) at New York University. The 198 subjects in this experiment were recruited from undergraduate classes at New York University and had no previous experience in our experiments. In each session, after the subjects read the instructions, the instructions were also read aloud by an experimental administrator.[12] Each session lasted for about 90 minutes and each subject participated in only one session. An \$8 participation fee and subsequent earnings, which averaged about \$21, were paid in private at the end of the session. Throughout the experiment, we ensured anonymity and effective isolation of subjects in order to minimize any interpersonal influences that could stimulate a specific pattern of behavior.[13] The experiment was computerized and it was programmed in z-Tree (Fishbacher [29]). Appendix B contains the instructions for one of the sessions.

We report two treatments: SIM and SEQ. For each treatment, we ran four different sessions in which we varied the informativeness of subjects' signals. In each session, subjects played the specified game for 40 independent rounds. Subjects were randomly matched with another subject in the laboratory at the beginning of each round.

In our two treatments, the precise game that subjects played was a special case of the problem that we analyzed theoretically. The game was simplified as follows. First, the payoffs were altered in order to ensure that a subject's final payoff was non-negative. Second, the distribution of the state variable Θ was uniform over the support $[20, 50]$. In addition, conditional on the state θ, each subject i received an independent signal x_i which was drawn from a uniform distribution over the support $[\theta - e, \theta + e]$, where $e \geq 0$ parametrizes the informativeness of signals. Finally, in the SEQ treatment the length of the game was set to be $\tau = 3$.[14]

---

[12] At the end of the first round, the subjects were asked if there was anything they did not understand. No subject reported any problems with understanding the procedures or using the computer program.

[13] Participants' workstations were isolated by cubicles, making it impossible for participants to observe others' screens or to communicate. We also made sure that all the participants remained silent throughout the session. At the end of a session, participants were paid in private according to the number on their workstation.

[14] Recall that, in the equilibrium of the SEQ game, either a player never invests or he invests *the latest* in the second period. Allowing the subjects to make their investment decisions in three periods, enabled us to test whether this property of the equilibrium empirically holds.

TABLE 1: SUMMARY OF TRETAMENTS

Treatment	Number of Subjects	c	e	Number of Rounds
SEQ	22	2	0	40
SEQ	24	2	2	40
SEQ	28	2	5	40
SEQ	32	2	10	40
SIM	16	NA	0	40
SIM	16	NA	2	40
SIM	24	NA	5	40
SIM	36	NA	10	40

Table 1 provides a summary of our treatments. As we mentioned above, the parameter e indicates the noisiness of signals. Also, c indexes the per-period cost of delayed investment. Throughout the paper we will write SEQ(c, e) and SIM(e) to refer to the treatments with corresponding parameters c and e.

SEQUENTIAL (SEQ) In each session, subjects played the game in Figure 3. For each of the four different SEQ sessions, subjects faced a different amount of noise, e, as shown in Table 1. Each round consisted of three periods. In each period, subjects simultaneously chose I or W. Choosing I was irreversible, while choosing W was reversible in the first two periods. If a subject chose W in the first period, then in the second period he would observe the first-period decision of his match. Similarly, in the third period, if the subject chose W in period one or two, he would observe his match's previous decisions. The cost of investment was only incurred if the subject chose I in period 1 or 2, after having previously chosen W. The cost of investment in the period 2 was 2 points, while it was 4 points in period 3. Immediate investment was costless.

FIGURE 3: THE SEQ GAME
(IGNORING COSTS OF DELAYED INVESTMENT)

	I	W
I	θ, θ	$\theta - 20, 25$
W	$25, \theta - 20$	$25, 25$

SIMULTANEOUS (SIM) In the SIM sessions the payoffs were identical to those in the SEQ treatment. The only exception was that each round of SIM consisted of a single period in which subjects could make their decision.

Throughout the experiment we used a neutral language and replaced the terms "investing" and "waiting" with "action A" and "action B", respectively.

5.2 BASIC TESTS

In order to find out whether the observed behavior is compatible with the basic predictions of the theory, in Table 2 we first focus on three simple, yet fundamental, tests that question subjects' behavior. The first test is about whether subjects respect dominance: Given the parameters of our experiment, if a subject's signal is below $25 - e$ and above $45 + e$ then waiting and investing are dominant strategies, respectively. Our subjects violated dominance only 4.8% of the time when one's estimate was in the dominance region in the SEQ treatment and only once in the SIM treatment. The second test concerns the timing of the investment decision in the SEQ game. In the equilibrium, a player either never invests, invests in the first period or invests in the second period, after observing the other player invest in the first. This is an important property of the equilibrium, and its frequent violation would imply a basic misunderstanding of strategic interaction in our game. In the SEQ treatment, in only 0.47% of the observations did a subject invest in period 3. Finally, a subject should never invest in period 2 after having observed that his match *did not* invest in period 1. We observe this type of behavior in only 1.05% of all the cases in which a subject who did not invest in period 1 observes that his match did not invest in period 1 either.

TABLE 2: BASIC TESTS OF THE THEORY

	SEQ	SIM
Violations of dominance	32 (664)	1 (472)
Invest in period 3	20 (4240)	NA
Invest in period 2 despite observing W in period 1	10 (954)	NA

The numbers in parentheses indicate the relevant sample sizes.

Of this small number of errors, approximately half took place in the first 10 rounds and 80% in the first 20 rounds. This is an indication that not only are these errors rare, but they also vanish as the subjects become more familiar with the experiment. Since the number of violations are extremely low we can confidently conclude that the subjects in our experiment had a strong grasp of fundamental aspects of the strategic problem they were faced with. Given this, we can delve more deeply into more specific aspects of behavior using theory as a guide.

5.3 SUBJECTS' BEHAVIOR IN PERIOD ONE

The thresholds that characterize the equilibrium of SIM and SEQ games are the ideal tool to analyze subjects' behavior. Therefore, we first focus on the decisions of the subjects in the first period by analyzing the thresholds at the aggregate level. In order to do this, for each session, we use the logistic distribution to fit investment decisions in period one. That is, we write the probability of investing in period one conditional on observing a signal x as

$$\Pr(\textsc{i}|x) = \frac{\exp(a+bx)}{1+\exp(a+bx)}. \tag{4}$$

TABLE 3: ESTIMATED PERIOD ONE THRESHOLDS (LAST 20 ROUNDS)

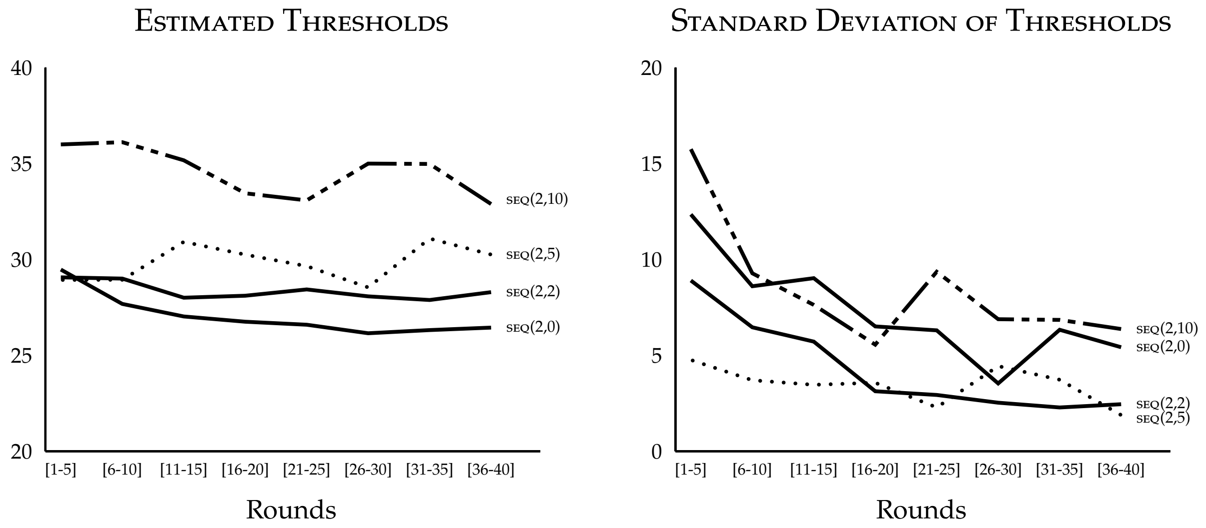
Treatment	a	b	Mean Threshold	S.D. of Threshold	Equilibrium Threshold	p-value (theory)
SEQ$(2,0)$	-8.61 (-2.56)	0.33 (2.48)	26.42	5.57	27.00†	0.40
SEQ$(2,2)$	-19.73 (-6.92)	0.70 (6.95)	28.35	2.61	27.79	0.29
SEQ$(2,5)$	-17.29 (-5.31)	0.59 (5.46)	29.25	3.07	28.92	0.51
SEQ$(2,10)$	-8.06 (-5.47)	0.24 (5.66)	33.09	7.45	30.74	0.02
SIM(0)	-55.14 (-4.34)	2.17 (4.22)	25.46	0.84	35.00	0.00
SIM(2)	-27.66 (-6.35)	0.93 (6.21)	29.61	1.94	35.00	0.00
SIM(5)	-19.94 (-4.97)	0.65 (4.77)	30.67	2.79	35.00	0.00
SIM(10)	-13.20 (-8.05)	0.41 (8.49)	32.29	4.44	35.00	0.00

Robust z-statistics are in parentheses (clustering at the subject level).
*** significant at 1%; ** significant at 5%; * significant at 10%.
† There are actually multiple equilibria of the SEQ$(2,0)$ game. A threshold of 27 represents the limiting equilibrium as $e \rightarrow 0$.

The ratio $-a/b$ is the mean threshold, while $\pi/b\sqrt{3}$ is the standard deviation of the estimated threshold. The second and third columns of Table 3 reports estimation results of (4) for all sessions. Robust z-statistics, which correct for clustering at the subject level, are reported below the estimated coefficients in parentheses. The estimated mean thresholds and their standard deviations are in the fourth and the fifth column, respectively. The equilibrium thresholds are provided in the sixth column. The last column reports $p-$value of the null hypothesis that the estimated threshold is equal to the theoretical threshold.

We report the estimation results for the last 20 rounds because we find that the subjects becomes more experienced as they play more rounds. In order to see this, we estimate the same logit regression (4) for each five-round increments. Figure 4 plots the estimation results for the seq treatment (results are similar for the sim treatment). As the left panel of Figure 4 depicts estimated thresholds seem to be constant over time.[15] However, in all sessions, the standard deviation of estimated thresholds have a fairly pronounced downward trend.[16] This suggests that while subjects' thresholds do not change much over time, they appear to become sharper in later rounds. Therefore, by focusing on the final 20 rounds, we are more likely to detect any deviations from the theoretical benchmarks.

Figure 4: Estimated Period One Thresholds & S.D. of Thresholds in the seq Treatment (5-Round Increments)



The estimation results readily suggest that the mean threshold is close to the equilibrium threshold in all sessions. In fact, except for seq(2,10), we fail to reject the hypothesis that the mean threshold is the same as the theoretical threshold in all sessions. In the seq(2,10) session the mean threshold is higher than the equilibrium threshold.

---

[15] Indeed, comparing any two consecutive five-round increments, a Wald test never rejects the null hypothesis that the thresholds are the same. In all cases, $p > 0.11$.

[16] In all cases, a Wald test indicates that the standard deviation of the estimate threshold is significantly smaller over the final 20 rounds than over the initial 20 rounds. In all cases, $p < 0.05$.

In terms of comparative statics, estimated thresholds comply with the theoretical predictions. We test this formally by pooling the data. Let $\hat{k}^1(c,e)$ denote the estimated period-one threshold of the session SEQ(c,e). From a series of Wald tests, we find that

$$\hat{k}^1(2,0) \underset{p=.023}{<} \hat{k}^1(2,2) \underset{p=.20}{=} \hat{k}^1(2,5) \underset{p<.01}{<} \hat{k}^1(2,10).$$

Therefore, in all cases, our empirical results have the directionally correct comparative static and in two of the three cases, the result is statistically significant. Overall, Table 3 provides fairly strong support that period-one behavior in the SEQ sessions is consistent with the theory.

In the SIM treatment, as can be seen in Table 3, we observe that the estimated thresholds are significantly below the equilibrium thresholds in all sessions. This result conforms to the results of Heinemann, Nagel, and Ockenfels [37] and Duffy and Ochs [21], who find that subjects's thresholds are lower than theoretical thresholds. Because of this, as we will presently show, subjects in our SIM treatments had higher earnings than the theoretical prediction.

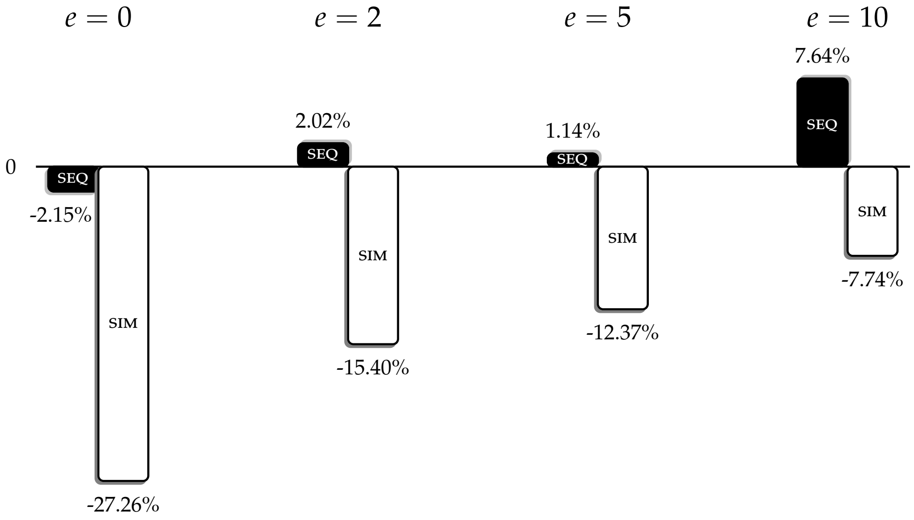
In order to summarize our findings, Figure 5 compares the two treatments in terms of the percentage deviations of the estimated thresholds from the theoretical thresholds in all sessions: while the estimated thresholds are consistently below the theoretical thresholds in the SIM sessions, in the SEQ sessions the opposite seems to be true. This strongly indicates that in SIM, subjects invests more often than the predictions of the theory; hence, coordinate on the efficient outcome more often. On the other hand, while the estimated threshold is very close to the theoretical threshold, we also saw in Table 3 that the thresholds appear to be less precise than in SIM. Moreover, in order to be able to comment on the efficiency of the outcomes we need to analyze subjects' behavior in the second period. This is the purpose of the next section.

5.4 Subjects' Behavior in Period Two

In order to understand subjects' behavior in period two, we run the logit regression (4), constraining the data to those subjects who did not invest in period one, but observed that their match invested. Table 4 reports the results in the same structure as Table 3. Again we report the results for the behavior in the last 20 rounds since similar learning effects still exist. For example, in the SEQ(2,10) treatment, the standard deviation of the estimated threshold went from 18.2 in the first 20 rounds to 9.7 in the last 20 rounds.

In the second period we observe that the mean thresholds are not below the theoretical values. Specifically, for $e \in \{0,2\}$, a Wald test cannot reject that the period-two threshold is identical to the theoretical prediction ($p_0 = 0.34$ and $p_2 = 0.15$, respectively), while

Figure 5: Percentage Deviation of Estimated Thresholds from Theoretical Thresholds



for $e \in \{5, 10\}$, we reject the null hypothesis ($p_5 = 0.025$ and $p_{10} = 0.019$, respectively). This observation suggests that observing an investment in the first period triggered more investments in the second period, which contributes to coordination on the efficient outcome.

The fact that the estimated thresholds are above the theoretical thresholds suggests that some of the subjects who invested in the second period, should have already invested in the first period had they followed the theoretical thresholds. Since the estimated thresholds do not inform us about the composition of the subjects who invested in the second period, they are not sufficient to compare the coordination rates in the SIM and SEQ treatments. Clearly the number of the second-period investors, as well as the distribution of their signals are important to compare the two treatments and understand the effect of subjects' timing decision on coordination, and in turn, on welfare. To this end, next, we look at the actual outcomes that we observed in the experiment.

5.5 OUTCOMES

Although our analyses of the first- and the second-period behavior shed light on the general pattern of subjects' behavior, it is not sufficient to compare coordination rates across treatments. On the one hand, low estimated thresholds indicate that subjects are well inclined to coordinate in the SIM sessions vis-à-vis the SEQ sessions. On the other hand, in the SEQ sessions, investment in the first period seems to trigger more investment in the second period. In order to clarify this issue we compare actual outcomes that we observe in all sessions.

Table 5 reports the observed frequencies of each of the four equilibrium paths for each

Table 4: Estimated Period-two Thresholds Conditional Upon Observing Investment in Period One (last 20 Rounds)

Treatment	a	b	Mean Threshold	S.D. of Threshold	Equilibrium Threshold	p-value (theory)
seq$(2,0)$	-94.97 (-2.43)	3.55 (2.48)	26.72	0.51	27.00	0.34
seq$(2,2)$	-6.65 (-1.38)	0.22 (1.22)	30.79	8.40	25.61	0.15
seq$(2,5)$	-9.15 (-1.77)	0.29 (1.54)	31.18	6.18	23.54	0.03
seq$(2,10)$	-4.56 (-3.13)	0.19 (3.58)	24.45	9.74	20.13	0.02

Robust z-statistics are in parentheses (clustering at the subject level).
*** significant at 1%; ** significant at 5%; * significant at 10%.

session: (*i*) (I_1, I_1): both subjects invest in period one; (*ii*) (I_1, I_2): one subject invests in period 1 and the other invests in period 2; (*iii*) (I_1, W): one subject invests in period 1 and the other subject never invests; and (*iv*) (W, W): neither subject invests. The final column reports the frequency of any other type of outcome, such as investing in period 3 or investing in period 2 after having observed W. Below each reported frequency, in brackets, we also report the expected frequency of each outcome if subjects were using the correct theoretical benchmark.

Not surprisingly, in the sim treatments, we observe the outcome (I_1, I_1) more frequently than theory predicts. In the seq treatment, we have an opposite result: The observed frequency of (I_1, I_1) is (often substantially) below the expected frequency. Again, this is consistent with our earlier finding that subjects used a higher threshold than predicted by theory. In order to compare the frequencies of the outcomes in which subjects successfully coordinate on investment, in the seq treatment, we have to sum over the first two columns. Then we obtain the frequency that subjects *eventually* coordinated on investment. These figures make it apparent that subjects eventually coordinated on investment more frequently in seq than in sim, with the lone exception occurring when $e = 0$. Indeed, subjects coordinated on the efficient outcome in 76.36 and 76.25 percent of the times in seq(2,0), and sim(0), respectively.

The observation that the outcomes are seemingly the same in seq(2,0) and sim(0) is quite surprising since it suggests that the complementarity effect of timing does not play a significant role in coordination. Perhaps, an even more puzzling observation is that, some subjects who would have invested in the sim(0) session, preferred to delay and invested

TABLE 5: ACTUAL VS. PREDICTED FREQUENCIES OF OUTCOMES

Treatment	$(\mathrm{I}_1, \mathrm{I}_1)$	$(\mathrm{I}_1, \mathrm{I}_2)$	$(\mathrm{I}_1, \mathrm{W})$	(W, W)	OTHER
SEQ(2,0)	65.91 [77.27]	10.45 [NA]	7.27 [NA]	15.68 [22.73]	0.68 [NA]
SEQ(2,2)	65.00 [72.08]	6.04 [6.04]	7.50 [0.42]	19.58 [21.46]	1.88 [NA]
SEQ(2,5)	62.68 [66.43]	5.00 [7.86]	8.75 [0.89]	22.68 [24.82]	0.89 [NA]
SEQ(2,10)	38.12 [54.06]	19.22 [18.45]	13.44 [2.66]	27.34 [24.84]	1.88 [NA]
SIM(0)	76.25 [50.31]	NA [NA]	8.12 [NA]	15.62 [49.69]	NA [NA]
SIM(2)	62.19 [44.38]	NA [NA]	11.88 [4.06]	25.94 [51.56]	NA [NA]
SIM(5)	57.71 [46.25]	NA [NA]	15.42 [12.29]	26.88 [41.46]	NA [NA]
SIM(10)	45.83 [39.44]	NA [NA]	27.64 [21.11]	26.53 [39.44]	NA [NA]

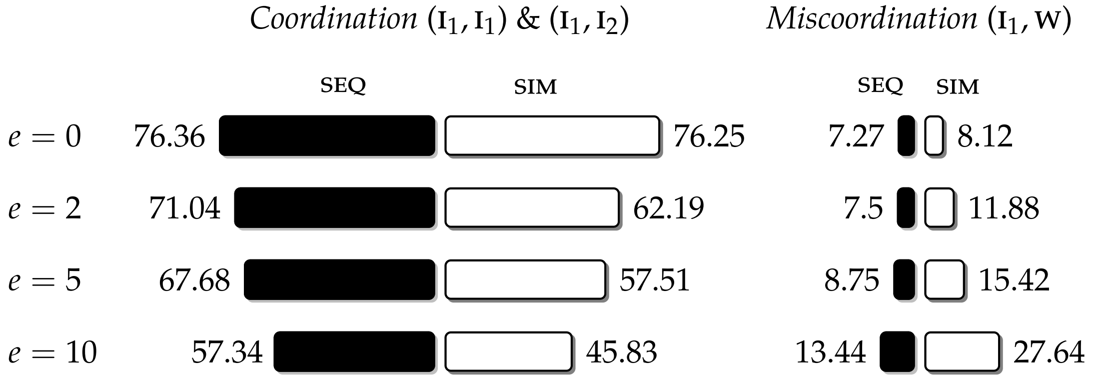
The numbers in brackets represent the frequencies that would have occurred if subjects were using the equilibrium thresholds. NA indicates that the event is either not a possible outcome or off-the-equilibrium path.

in the second period in the SEQ(2,0) session despite the cost. In an independent study Duffy and Ochs [21] find similar results and conclude that looking at the data one cannot distinguish between static and dynamic games under complete information. Our results confirm these findings.

Figure 6 reorganizes coordination and miscoordination rates from Table 6 and summarizes the comparison between SEQ and SIM. Note that $e = 0$ is the only case in which SIM and SEQ are indistinguishable. The same result is not true under asymmetric information where we observe that subjects coordinate better in SEQ sessions than in SIM sessions. Moreover, it seems the difference in coordination rates between SEQ and SIM increase in e—an issue that we will discuss in the next section. When we look at the frequencies of miscoordination, $(\mathrm{I}_1, \mathrm{W})$, we observe a similar pattern: This sort of coordination failure is less frequent in SEQ than SIM, and the difference increases as asymmetric information becomes more pronounced. This suggests that the learning effect of timing is significant.

Thus, under asymmetric information, timing enhances coordination — predominantly through the learning effect. The fact that coordination is less frequent than the theory can be explained by the failure of complementarity effect of timing. In order to complete the

Figure 6: Coordination & Miscoordination Rates



Coordination (I_1, I_1) & (I_1, I_2)

Miscoordination (I_1, W)

comparison between SIM and SEQ we must also study how this is all reflected in the actual payoffs and whether welfare improves.

5.6 Welfare

The theoretical results suggest that as the noise, e, approaches zero the equilibrium of the SEQ game becomes fully efficient, while in SIM the equilibrium is not necessarily efficient for relatively smaller states. Moreover, the difference in average welfare between the two treatments is increasing in e. That is, as noise increases, endogenous timing becomes more valuable. In this section we will analyze the effect of endogenous timing on welfare and test these hypotheses.

For the analysis that follows we introduce some more notation. Let π^a denote the actual payoff received by a subject, including any costs due to delayed investment, and $\pi^{max} := \max\{25, \theta\}$ denote the ex post efficient payoff; i.e., both subjects invest in the first period if and only if $\theta \geq 25$. Also, let π^{eq} denote the payoff that a player would have received if subjects played according to the theoretical prediction.

For each of our treatments, Table 6 reports the ratios π^a/π^{max} and π^a/π^{eq}. Looking first at the left side of the table where we report π^a/π^{eq}, one can see that subjects in the SIM treatments actually earned more, on average, than the equilibrium payoff. This is because, subjects in these treatments used lower thresholds than the global games theory predicts. In exactly the opposite way, subjects in the SEQ treatments generally earned less than their equilibrium payoff because the subjects used higher thresholds than predicted by the theory.

The middle columns of Table 6 report π^a/π^{max}. Here we obtain a result that conflicts with Corollary 1's prediction that, in the limit as noise disappears, subjects in the SEQ game should earn more than subjects in the SIM game. Indeed, as we have discussed,

Table 6: Efficiency Ratios

Parameters	π^a / π^{eq}		π^a / π^{max}		Comparison	
(c, e)	SIM	SEQ	SIM	SEQ	SEQ − SIM	PREDICTED
$(2, 0)$	1.039	0.965	0.966	0.963	-0.003	0.059
$(2, 2)$	1.037	0.969	0.944	0.959	0.015^{*}	0.063
$(2, 5)$	1.05	0.970	0.923	0.949	0.026^{***}	0.073
$(2, 10)$	1.02	0.957	0.862	0.905	0.043^{***}	0.096

† Although there is no cost in the SIM treatments, we report estimates in the row that corresponds to the appropriate value of e.
* 10% level of significance; *** 1 % level of significance. H_0 : SEQ − SIM = 0.

when $e = 0$, the SEQ and SIM treatments are virtually indistinguishable from each other. However, consistent with our theory, for $e \geq 2$, we find that earnings are significantly higher in the SEQ treatment than in the SIM treatment.[17] One can also see that as noise increases, the difference in efficiency between SEQ and SIM increases, though not by as much as the theory predicts. In particular, if we estimate the following model via OLS,

$$\pi^a / \pi^{max} = \beta_0 + \beta_1 e + \beta_2 e \cdot \mathbb{D}_{\text{SEQ}} + \beta_3 \mathbb{D}_{\text{SEQ}} + \epsilon,$$

where $\mathbb{D}_{\text{SEQ}}$ is the dummy variable that takes value 1 if it is a SEQ session, and 0 otherwise, we obtain $\beta_1 < 0$, $\beta_2 > 0$, both of which are significant at the 1% level. Thus, welfare declines as noise increases but, consistent with theory, it declines significantly more slowly in the SEQ treatment.

5.7 Why Do Subjects Delay?

The lesson we have learned from the analysis of thresholds is that while the theory is consistent with the behavior of the subjects who invest in the first period, it under-predicts the average thresholds of those who delay investment. In this section, we will analyze the behavior of the subjects who delay investment and seek an explanation for their behavior.

As we have discussed extensively in the theory section delaying investment has two advantages. First, a player gains information through the other player's investment decision. Second, since complementarities are present, by delaying investment a player can avoid the risk of investing alone. On the other hand, delayed investment comes with the direct cost, c, and the indirect cost (from not investing immediately) of not sending a positive signal about the state. For high enough signals (*i.e.*, $x \geq k^1$), this trade-off results in immediate investment, while for intermediate signals (*i.e.*, $k^2 \leq x < k^1$) it leads to a

---

[17] Letting t^e_{df} denote the t statistic when noise is e and the degrees of freedom are df, we have that $t^0_{36} = 0.26$ ($p = 0.800$), $t^2_{38} = 1.77$ ($p = 0.084$), $t^5_{50} = 3.25$ ($p = 0.002$) and $t^{10}_{66} = 4.43$ ($p = 0.000$).

conditional investment strategy. Finally, for small enough signals (*i.e.,* $x < k^2$), a subject will never invest.

TABLE 7: THE FREQUENCY OF DELAYED INVESTMENT AFTER OBSERVING I
(CONDITIONAL ON W IN PERIOD 1)

Treatment	$x < k^2$	$k^2 \leq x < k^1$	$x \geq k^1$	UNCONDITIONAL	N
SEQ$(2,0)$	9.68%	NA	92.00%	60.49%	81
SEQ$(2,2)$	0.00%	25.00%	72.50%	49.30%	71
SEQ$(2,5)$	16.67%	29.73%	56.67%	37.97%	79
SEQ$(2,10)$	20.00%	46.05%	78.07%	60.00%	215

In Table 7 we report the frequency with which subjects invested with some delay, after having observed that the other subject invested in the first period. For instance, in the SEQ(2,5) session, out of all subjects who received a signal between k^2 and k^1, 29.73 percent invested in period two or three.

In theory, we should only see period-two investment in the region $k^2 \leq x < k^1$; however, as we can see in the fourth column, this is not the case. Indeed, there are many instances in which $x \geq k^1$, but the subject chose to delay. Form the theory perspective, these are the subjects who should have invested in the first period. Hence, their behavior seems to be motivated by an aversion to strategic uncertainty about the other subjects' decision. In fact, a large fraction of subjects whose signals exceed k^1 invested once they observe other subject's investment.

Of course, we have seen an even stronger evidence that some subjects seek confidence by delaying investment decision despite their high signal in the case of SEQ$(2,0)$, where subjects actually have *complete* information about the state. As Table 5 indicated, 10.45% of outcomes were of the form $(\mathrm{I}_1, \mathrm{I}_2)$, meaning that one of the subjects delayed investment. This is in sharp contrast to the theoretical prediction that all investment should take place immediately. Thus, some subjects at least, would appear to be excessively cautious, not wanting the risk the chance of miscoordination.

When we look at the subjects whose signals are between k^2 and k^1, there seems to be an under-appreciation of the information revealed by the decision of the other subject. While those subjects for which $k^2 \leq x < k^1$ were correct to initially delay, the fact that the frequency of delayed investment is relatively low suggests that they did not sufficiently internalize the signal that they received from observing their match's investment. However, note that, these subjects invest more often as the noise e increases. In other words, while the value of the information increases, subjects better appreciates the value of information inherent in investment decisions.

These two observations suggest that there are two possible types of behavior that may explain the data: the rational type that we have analyzed in the theory section and the type that seeks confidence in order to invest.

5.7.1 Confidence Seekers

Given the results that we have presented thus far, we conjecture that there are some subjects that we will call *confidence seekers*. In our experiment, confidence seekers are those subjects who, upon receiving a signal for which immediate investment is optimal, prefer to wait to observe the decision of their match. Confidence seekers have been shown to exist in other experiments. For example, Eliaz and Schotter [22] show that many subjects are willing to pay for information even though the *ex ante* optimal action remains optimal *regardless* of the information revealed.

We feel that confidence seeking behavior may express itself in a couple of distinct ways. First, our theory predicts that the period one threshold of SEQ should be below the threshold of SIM. To the extent that this is not true, it is suggestive of confidence seeking. That is, confidence seeking subjects in SEQ may prefer to use the same (or possible higher) threshold than in SIM in order to gain information about the state and their match's decision. On this front, recall Table 3 and look at the estimated thresholds. As can be seen, for $e \in \{0, 10\}$, the threshold is actually higher in the SEQ game than in the SIM game. For $e \in \{2, 5\}$, the estimated thresholds are lower in SEQ; however, the significance levels are only 8% and 6%, respectively. Thus, thresholds are not consistently lower in SEQ than in SIM, which provides some suggestive evidence in favor of our confidence seekers hypothesis.

Second, observe that while we would expect subject-specific heterogeneity in both the SEQ and SIM treatments, only in the SEQ treatment, where subjects have the opportunity to delay investment, can confidence seeking behavior be expressed. Therefore, to the extent that there is greater variation in behavior in the SEQ treatment than in the SIM treatment, this is further indication that such behavior may explain our results. To this end, recall again Table 3 and specifically look at the standard deviation of the estimated thresholds. As can be seen, for all values of e, the standard deviation is higher in the SEQ treatment than in the corresponding SIM treatment. Furthermore, the difference is statistically significant for $e \in \{0, 10\}$. Again, this result further reinforces our belief that confidence seekers may be present in our data, at least for $e \in \{0, 10\}$.

To gain further insights into the possible presence of confidence seekers in our SEQ treatment, we estimate a simple two-type mixture model. In particular, for the first type, τ_1, we assume that the probability of investing in period one, conditional on observing a signal x is

$$\Pr(\text{I}|x,\tau_1) = \frac{\exp(a+bx)}{1+\exp(a+bx)}. \tag{5}$$

For the second type, τ_2, we assume that the probability of investing in period one, conditional on observing a signal x is given by

$$\Pr(\text{I}|x,\tau_2) = \frac{\exp(a+a'+bx)}{1+\exp(a+a'+bx)}. \tag{6}$$

That is, the mean threshold for the first type is simply $-\frac{a}{b}$, while the threshold of the second type is *shifted* by an amount $-\frac{a'}{b}$ to $-\frac{a+a'}{b}$. If our conjecture about confidence seekers is correct, then we would expect there to be a small, but significant, group of type 2 subjects for whom $a' < 0$, indicating a rightward shift in the mean threshold.

Denote a generic type by τ. Then the likelihood for player i, conditional upon being type τ, can be written as

$$\mathcal{L}_i(\tau,\Lambda) = \prod_{t=1}^{T} \Big(\Pr(\text{I}|x,\tau,\Lambda)\Big)^{y_{i,t}} \Big(1-\Pr(\text{I}|x,\tau,\Lambda)\Big)^{1-y_{i,t}},$$

where $y_{i,t} = 1$ if subject i invests in period one of round t and 0 otherwise, and Λ is the parameter vector that will be estimated.

Next, given the conditional likelihood for subject i, we may then easily write the unconditional likelihood function as

$$\mathcal{L}_i(\Lambda,\rho) = \rho\mathcal{L}_i(\tau_1,\Lambda) + (1-\rho)\mathcal{L}_i(\tau_2,\Lambda),$$

where $\rho \in [0,1]$ is the probability of type 1. Finally, taking the product over all subjects, i, we have the likelihood function

$$\mathcal{L}(\Lambda,\rho) = \prod_{i=1}^{N} \mathcal{L}_i(\Lambda,\rho),$$

which can then be maximized to obtain estimates for $\Lambda = (a,a',b)$ and ρ.

Table 8 reports the results of this exercise. In all sessions a likelihood ratio test easily rejects the null hypothesis that there is a single type, against the alternative of two types. Looking at the SEQ treatment, one can see that the type 1 threshold is below the equilibrium threshold, while the type 2 threshold is always above the equilibrium threshold. This is especially true for SEQ$(2,0)$ and SEQ$(2,10)$, by quite a large amount.

Although heterogeneity is present in both the SEQ and SIM treatments, it would appear to be greater in the former treatment. Specifically, let μ_i denote the threshold of Type i

TABLE 8: ESTIMATED THRESHOLDS FOR THE TWO TYPE MIXTURE MODEL
(LAST 20 ROUNDS)

Treatment	Equilibrium	Type 1	Type 2	ρ	$\frac{\pi}{b\sqrt{3}}$	$p-$value
SEQ$(2,0)$	27.00	25.591	35.896	0.909	3.439	0.000
SEQ$(2,2)$	27.79	26.146	30.016	0.497	1.745	0.000
SEQ$(2,5)$	28.92	25.443	30.389	0.247	2.341	0.000
SEQ$(2,10)$	30.74	29.714	38.565	0.632	5.647	0.000
SIM(0)	35.00	24.945	27.163	0.789	0.493	0.006
SIM(2)	35.00	28.091	30.971	0.489	1.309	0.000
SIM(5)	35.00	29.972	38.156	0.880	1.914	0.000
SIM(10)	35.00	29.455	34.799	0.504	3.336	0.000

and $\bar{\mu}$ denote the average threshold. Then, we measure heterogeneity as the standard deviation: $\sigma = \sqrt{\rho(\mu_1 - \bar{\mu})^2 + (1-\rho)(\mu_2 - \bar{\mu})^2}$. With the exception of $e = 5$, the standard deviation, σ, is between 44 and 228% higher in SEQ$(2, e)$ than in SIM(e).[18] Thus it appears that endogenous timing leads to increased heterogeneity. Moreover, the precision of the thresholds, $\frac{\pi}{b\sqrt{3}}$, is lower by between 22 and 598% in the SEQ treatments than in the corresponding SIM treatments. Therefore, not only is there more variation in thresholds in SEQ than in SIM, the thresholds themselves are less precise.

REMARK 1. *It is difficult to speak of the statistical significance of the results comparing σ and $\frac{\pi}{b\sqrt{3}}$ across* SEQ *and* SIM*. An alternative approach is to estimate thresholds for each individual and then to ask (i) is there greater variation in individual thresholds in* SEQ*? and (ii) given the estimated individual thresholds, are subjects more prone to making mistakes in* SEQ*?*[19] *The answer to the first question is yes, there is greater variation in estimated individual thresholds for $e \in \{0, 10\}$ (in both cases, $p < 0.01$). In answer to the second question, subjects make significantly more mistakes in* SEQ *than in* SIM *for $e \in \{0, 2\}$ (in both cases, the rank sum test gives $p < 0.05$, while for $e = 10$, the p-value is 0.107). Furthermore, the variance in the number of mistakes is significantly higher in* SEQ *for $e \in \{2, 10\}$ ($p < 0.01$). Finally, the average of individual thresholds is only significantly lower in* SEQ$(2,5)$ *than in* SIM(5)*. Indeed, when $e = 0$, there is actually some evidence that thresholds are higher in* SEQ *(rank-sum test, $p = 0.097$). Thus, we are quite confident that there is much greater heterogeneity in the* SEQ *treatment than in the* SIM *treatments, which we attribute to subjects using the ability to delay investment in the* SEQ *treatment in order to gain confidence.*

---

[18] For $e = 5$, the standard deviation is 20% lower in the SEQ treatment.

[19] Given the estimated threshold, we define a mistake as either (i) a subject does not invest despite receiving a signal higher than the threshold or (ii) a subject invests despite receiving a signal lower than the threshold.

6 CONCLUDING REMARKS

The present paper analyzed a simple global coordination game with endogenous timing in order to understand the effect of timing on coordination under asymmetric information. The theoretical analysis highlights two forces behind timing decisions: learning and complementarities. We show that in the equilibrium optimistic players invest first and convey information about the profitability of investment. In addition to this learning effect, the investment eliminates the strategic uncertainty and encourages investment through strategic complementarities. The normative analysis show that endogenous timing leads to an efficient outcome especially as players' private information become infinitely informative. Moreover, the relative effect of endogenous timing vis-à-vis simultaneous case on welfare increases as signals become less informative.

The theoretical analysis provides us with interesting predictions that are testable in the lab. This is what we do in the experimental exercise. Our experiment was designed with two questions in mind: (*i*) how do the two effects of timing—learning and complementarities—determine subjects' actual behavior; and (*ii*) how does the option to delay affect welfare?

A summary of the main results of the experiment is as follows. First, we find that compared to theory benchmark, in the simultaneous treatments subjects invest more whereas in the sequential treatments they invests less. Furthermore, we also show that the comparative statics on average thresholds are largely consistent with the theoretical predictions.

We also find that coordination rates are higher in the sequential treatments than the simultaneous ones especially when the signals are less informative. In fact, we find that the complementarity effect of timing is not significant while it seems there is a significant learning effect. Overall, endogenous timing increases efficiency in the same manner that the theory predicts. The significant efficiency gain that we observe in the experiment makes us argue that the behavioral difference between static and dynamic global investments games is sufficiently different to justify a continued focus on behavior in dynamic games, especially in environments with asymmetric information.

REFERENCES

[1] Angeletos, G-M., C. Hellwig, and A. Pavan (2007). "Dynamic Global Games of Regime Change: Learning, Multiplicity, and the Timing of Attacks." *Econometrica* 75(3): 711-756.

[2] Alesina, A. and R. Perotti (1996). "Income Distribution, Political Instability and Investment." *Europen Economic Review* 40: 1203-1228.

[3] Aizenman, J. and N. P. Marion (2004). "The Merits of Horizontal Versus Vertical FDI in the Presence of Uncertainty." *Journal of International Economics* 62: 125-148.

[4] Angeletos, G-M. and I. Werning (2006). "Crises and Prices: Information Aggregation, Multiplicity, and Volatility." *American Economic Review* 96(5): 1720-1736.

[5] Banerjee, A (1992). "A Simple Model of Herd Behavior." *Quarterly Journal of Economics* 107(3): 797-817.

[6] Bikhchandani, S., D. Hirshleifer and I. Welch (1992). "A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades." *Journal of Political Economy* 100(5): 992-1026.

[7] Bolton, P. and J. Farrell (1990). "Decentralization, Duplication, and Delay." *Journal of Political Economy* 98(4): 803-826.

[8] Busse, M. and C. Hefeker (2007). "Political Risk, Institutions and Foreign Direct Investment." *European Journal of Political Economy* 23: 397-415.

[9] Cushman, D.O. (1985). "Real Exchange Rate Risk, Expectations, and the Level of Investment." *Review of Economics and Statistics* 67: 297-308.

[10] Choi, J. (1997). "Herd Behavior, the "Penguin Effect," and the Suppression of Informational Diffusion: An Analysis of Informational Externalities and Payoff Interdependency." *RAND Journal of Economics* 28(3): 407-425.

[11] Chamley, C. and D. Gale (1994). "Information Revelation and Strategic Delay in a Model of Investment." *Econometrica* 62(5): 1065-1085.

[12] Cabrales, A., R. Nagel, and R. Armenter (2007). "Equilibrium Selection Through Incomplete Information in Coordination Games: An Experimental Study." *Experimental Economics* 10: 221-234.

[13] Caplin, A. and J. Leahy (1994). "Business as Usual, Market Crashes, and Wisdom After the Fact." *American Economic Review* 84(3): 548-565.

[14] Caplin, A. and J. Leahy (1998). "Miracle on Sixth Avenue: Information Externalities and Search." *Economic Journal* 108(446): 60-74.

[15] Creane, A. and K. Miyagiwa (2007). "Foreign Direct Investment and Cost Uncertainty: Correlation and Learning Effects." Mimeo.

[16] Conley, T. and C. Udry. "Social learning through networks: The adoption of new agricultural technologies in Ghana." *American Journal of Agricultural Economics*, 83: 668-673, 2001.

[17] Carlsson, H. and E. van Damme (1993). "Global Games and Equilibrium Selection." Econometrica 61(5): 989-1018.

[18] Costain, J., F. Heinemann, and P. Ockenfels (2007). "Multiple Outcomes of Speculative Behavior in Theory and in the Laboratory." Mimeo.

[19] Dasgupta, A. (2007). "Coordination and Delay in Global Games." *Journal of Economic Theory* 134(1): 195-225.

[20] Diamond D.W., P.H. Dybvig (1983). "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy* 91(3): 401-419.

[21] Duffy, J. and J. Ochs (2009). "Equilibrium Selection in Static and Dynamic Entry Games." Mimeo.

[22] Eliaz, K. and A. Schotter (2009). “Paying for Confidence: An Experimental Study of the Demand for Non-Instrumental Information." *Games and Economic Behavior* forthcoming.

[23] Engle-Warnick, J. and S. Laszlo (2008). “Learning-by-Doing in an Ambiguous Environment." Mimeo.

[24] Farrell, J. (1987). “Cheap Talk, Coordination, and Entry.” *Rand Journal of Economics* 18(1): 34-39.

[25] Farrell, J. and G. Saloner (1985). “Standardization, Compatibility, and Innovation.” *Rand Journal of Economics* 16(1): 70-83.

[26] Farrell, J. and G. Saloner (1986). “Installed Base and Compatibility: Innovation, Product Preannouncements, and Predation.” *American Economic Review* 76(5): 940-954.

[27] Farrell, J. and G. Saloner (1988). “Coordination through Committees and Markets.” *Rand Journal of Economics* 19: 235-252.

[28] Fehr, E. and O. Shurchkov (2008). “Coordination and Learning in Dynamic Global Games: Experimental Evidence.” Mimeo.

[29] Fishbacher, U. (2007). “z-Tree: Zurich toolbox for ready-made economic experiments.” *Experimental Economics* 10(2): 171-178.

[30] Frankel, D., S. Morris, and A. Pauzner (2003). “Equilibrium Selection in Global Games With Strategic Complementarities.” *Journal of Economic Theory* 108(1): 1-44.

[31] Gale, D. (1995). “Dynamic Coordination Games.” *Economic Theory* 5(1): 1-18.

[32] Goldberg, L. S. and C. D. Kolstad (1995). “Foreign Direct Investment, Exchange Rate Variability and Demand Uncertainty.” *International Economic Review* 36: 855-873.

[33] Goldstein, I. and A. Pauzner (2005). “Demand-Deposit Contracts and the Probability of Bank Runs." *Journal of Finance* 60(3): 1293-1327.

[34] Gul, F. and R. Lundholm (1995). “Endogenous Timing and the Clustering of Agents Decisions.” *Journal of Political Economy* 103(5): 1039-1066.

[35] Harsanyi, J. and R. Selten. (1988). *A General Theory of Equilibrium Selection in Games*, MIT Press, Cambridge, Mass.

[36] Heidhues, P. and N. Melissas (2006). “Equilibria in a Dynamic Global Game: the Role of Cohort Effects.” *Economic Theory* 28(3): 531-557.

[37] Heinemann. F, R. Nagel, and P. Ockenfels (2004). “The Theory of Global Games on Test: Experimental Analysis of Coordination Games With Public and Private Information.” *Econometrica* 72(5): 1583-1599.

[38] Jordaan, A. A. (2009). *Foreign Direct Investment, Agglomeration and Externalities: Empirical Evidence from Mexican Manufacturing Industries.* Ashgate Economic Geography Series.

[39] Karlin, S. (1968). *Total Positivity*. Stanford, California, Stanford University Press.

[40] Katz, M. L. and C. Shapiro (1985). “Network Externalities, Competition, and Compatibility.” *American Economic Review* 75(3): 424-440.

[41] Katz, M. L. and C. Shapiro (1986). “Technology Adoption in the Presence of Network Externalities.” *Journal of Political Economy* 94(4): 822-841.

[42] Levin, D. and J. Peck (2008). "Investment Dynamics With Common and Private Values." *Journal Of Economic Theory* 143: 114-139.

[43] Matsuyama, K. (1991). "Increasing Returns, Industrialization, and Indeterminacy of Equilibrium." *Quarterly Journal of Economics* 106: 617-650.

[44] Markusen, J.R. (2004). *Multinational Firms and the Theory of International Trade*. MIT Press, Cambridge, MA.

[45] Morris, S. (2008). "Global Games" in *The New Palgrave Dictionary of Economics*, edited by S. Durlauf and L. Blume; Palgrave Macmillan.

[46] Morris, S. and H. S. Shin (1998). "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks." *American Economic Review* 88(3): 587-597.

[47] Morris, S. and H. S. Shin, (2003). "Global Games: Theory and Applications" in *Advances in Economics and Econometrics* (Proceedings of the Eighth World Congress of the Econometric Society), edited by M. Dewatripont, L. Hansen and S. Turnovsky; Cambridge University Press.

[48] Morris, S. and H. S. Shin (2004). "Coordination Risk and the Price of Debt." *European Economic Review* 48: 133-153.

[49] Rodrick, D. (1991). "Policy Uncertainty and Private Investment in Developing Countries." *Journal of Development Economics* 36: 229-242.

[50] Schmidt C.W. and U. Broll (2009). "Real Exchange-Rate Uncertainty and US Foreign Direct Investment: An Empirical Analysis." *Review of World Economics* 145: 513-530.

[51] Shaked, M. and J. G. Shanthikumar (2007). *Stochastic Orders*, Springer, New York, NY.

APPENDIX

A. OMITTED PROOFS

Proof of Lemma 1. Let $\theta \geq \theta'$, $x \geq x'$ and assume that $f_{\mathsf{X}_i}(\cdot|\theta)$ satisfies MLRP for $i = 1, 2$.
(i) Therefore

$$\begin{aligned} \frac{f_{\mathsf{X}_i}(x|\theta)}{f_{\mathsf{X}_i}(x|\theta')} &\geq \frac{f_{\mathsf{X}_i}(x'|\theta)}{f_{\mathsf{X}_i}(x'|\theta')} \\ \frac{f_\Theta(\theta|x)}{f_\Theta(\theta'|x)} \frac{f_\Theta(\theta')}{f_\Theta(\theta)} &\geq \frac{f_\Theta(\theta|x')}{f_\Theta(\theta'|x')} \frac{f_\Theta(\theta')}{f_\Theta(\theta)}. \qquad \text{(by Bayes' rule)} \end{aligned}$$

Hence $f_\Theta(\cdot|x)$ satisfies MLRP.
(ii) Rewrite $f_{\mathsf{X}_j}(x_j|x_i)$ as

$$f_{\mathsf{X}_j}(x_j|x_i) = \int f_{\mathsf{X}_j}(x_j|\theta) dF_\Theta(\theta|x_i).$$

Recall that $f_\Theta(\cdot|x)$ and $f_{\mathsf{X}_j}(\cdot|\theta)$ satisfy MLRP. Since MLRP is preserved under convolution (see Karlin [39]), $f_{\mathsf{X}_j}(\cdot|x_i)$ also satisfies MLRP. □

Proof of Lemma 2. Assume that $f_{\mathsf{X}_i}(\cdot|\theta)$ satisfies MLRP for $i = 1, 2$.
(i) Since $f_\Theta(\cdot|x)$ satisfies MLRP, we have $\mathbf{E}[\Theta|x] \geq \mathbf{E}[\Theta|x']$ for $x \geq x'$. Let $k \in \mathbb{R}$. Observe that

$$\begin{aligned}
\mathbf{E}[\Theta|x_i, x_j \geq k] &= \int_k \mathbf{E}[\Theta|x_i, x_j = x] dF_{\mathsf{X}_j}(x|x \geq k) \\
&\geq \int_k \mathbf{E}[\Theta|x_i, x_j = k] dF_{\mathsf{X}_j}(x|x \geq k) \\
&= \mathbf{E}[\Theta|x_i, x_j = k].
\end{aligned}$$

By use of this observation we get

$$\begin{aligned}
\mathbf{E}[\Theta|x_i] &= F_{\mathsf{X}_j}(k|x_i)\mathbf{E}[\Theta|x_i, x_j \leq k] + \big(1 - F_{\mathsf{X}_j}(k|x_i)\big)\mathbf{E}[\Theta|x_i, x_j \geq k] \\
&= F_{\mathsf{X}_j}(k|x_i) \int^k \mathbf{E}[\Theta|x_i, x_j = t] dF_{\mathsf{X}_j}(t|t \geq k) + \big(1 - F_{\mathsf{X}_j}(k|x_i)\big)\mathbf{E}[\Theta|x_i, x_j \geq k] \\
&\leq F_{\mathsf{X}_j}(k|x_i) \int^k \mathbf{E}[\Theta|x_i, x_j = k] dF_{\mathsf{X}_j}(t|t \geq k) + \big(1 - F_{\mathsf{X}_j}(k|x_i)\big)\mathbf{E}[\Theta|x_i, x_j \geq k] \\
&= F_{\mathsf{X}_j}(k|x_i)\mathbf{E}[\Theta|x_i, x_j = k] + \big(1 - F_{\mathsf{X}_j}(k|x_i)\big)\mathbf{E}[\Theta|x_i, x_j \geq k] \\
&\leq F_{\mathsf{X}_j}(k|x_i)\mathbf{E}[\Theta|x_i, x_j \geq k] + \big(1 - F_{\mathsf{X}_j}(k|x_i)\big)\mathbf{E}[\Theta|x_i, x_j \geq k] \\
&= \mathbf{E}[\Theta|x_i, x_j \geq k].
\end{aligned}$$

Similar arguments prove cases (ii) and (iii). □

Proof of Lemma 3. Let $\tau \geq 2$, and $\sigma := (\sigma_i, \sigma_j)$ be a monotone strategy profile. Suppose that σ is a perfect Bayesian equilibrium of the game. Note that if $(a_i^t, a_j^t) = (\mathbf{W}, \mathbf{I})$, by irreversibility of investment, there is no information to be revealed by player j in subsequent periods. Moreover, because it is costly to delay investment, either $a_i^s = \mathbf{W}$ for all $s > t$ or $a_i^s = \mathbf{I}$ for all $\tau \geq s > t$. Therefore, we will write $h_{x,y}^t$ to denote the history that players observes at time t in the equilibrium path, where the subscript x (y) denotes how many periods prior to t player i (j) switched to action $\mathbf{I}$. x $(y) = 0$ means player i (j) still waits. Also, we will denote all the relevant information that is available from the history of actions h^t by ι^t.

We will proceed by backward induction. If player i observes $h_{0,1}^\tau$, he invests in period τ if and only if

$$\mathbf{E}\left[\Theta\middle|x_i, \iota^{\tau-1}, x_j \geq k_j^{h_{0,0}^{\tau-1}}\right] \geq (\tau - 1)c. \tag{7}$$

By the absolute continuity of the random variables, the left hand side of the inequality (7) is continuous and increasing in x_i. Hence, either there is a unique $k_i^{h_{0,1}^\tau} \in \mathbb{R}$ such that at $x_i = k_i^{h_{0,1}^\tau}$ (7) holds at equality, or $k_i^{h_{0,1}^\tau} = \infty$, i.e. player i does not invest for any x_i.

If player i observes $h_{0,0}^\tau$, he invests in period τ if and only if

$$\mathbf{E}\left[\Theta\middle|x_i, \iota^{\tau-1}, x_j < k_j^{h_{0,0}^{\tau-1}}\right] - F_{\mathsf{X}_j}\left(k_j^{h_{0,0}^\tau}\middle|x_i, \iota^{\tau-1}, x_j < k_j^{h_{0,0}^{\tau-1}}\right) \quad \geq \quad (\tau - 1)c. \tag{8}$$

Since the left hand side of the inequality (7) is larger than the left hand side of the inequality (8), by Lemma 2, we get $k_i^{h_{0,0}^\tau} \geq k_i^{h_{0,1}^\tau}$.

Now, we analyze player i's optimal behavior at time $\tau - 1$.

If player i observes $h_{0,0}^{\tau-1}$, his expected payoff from investment in period $\tau-1$ is

$$\mathbf{E}\left[\Theta|x_i,\iota^{\tau-1}\right]-F_{\mathsf{X}_j}(k_j^*|x_i,\iota^{\tau-1})-(\tau-2)c. \tag{9}$$

where $k_j^*=\min\left\{k_j^{h_{0,0}^{\tau-1}},k_j^{h_{1,0}^{\tau}}\right\}$. In order to analyze player i's ex ante expected payoff from waiting in period $\tau-1$, suppose that $x_i\geq k_i^{h_{0,0}^{\tau}}$. Since $k_i^{h_{0,0}^{\tau}}\geq k_i^{h_{0,1}^{\tau}}$, player i will invest in period τ after any history. Thus, his ex ante expected payoff is

$$\begin{aligned}\mathbf{E}[\Theta|x_i,\iota^{\tau-1}]-F_{\mathsf{X}_j}\left(k_j^{h_{0,0}^{\tau-1}}\middle|x_i,\iota^{\tau-1}\right)F_{\mathsf{X}_j}\left(k_j^{h_{0,0}^{\tau}}\middle|x_i,\iota^{\tau-1},x_j<h_{0,0}^{\tau-1}\right)-(\tau-1)c&\\=\mathbf{E}[\Theta|x_i,\iota^{\tau-1}]-F_{\mathsf{X}_j}\left(\min\left\{k_j^{h_{0,0}^{\tau-1}},k_j^{h_{0,0}^{\tau}}\right\}\middle|x_i,\iota^{\tau-1}\right)-(\tau-1)c&\quad(10)\end{aligned}$$

Since we already have $k_j^{h_{0,0}^{\tau}}\geq k_j^{h_{1,0}^{\tau}}$, (9) is larger than (10). Thus $k_i^{h_{0,0}^{\tau}}\geq k_i^{h_{0,0}^{\tau-1}}$, i.e. player i would have already invested in period $\tau-1$ for $x_i\geq k_i^{h_{0,0}^{\tau}}$.

We now consider the case in which $x_i<k_i^{h_{0,1}^{\tau}}$. Since player i does not invest in period τ for $x_i<k_i^{h_{0,1}^{\tau}}$ his ex ante expected payoff from waiting is zero. Moreover the inequality (7) does not hold, which implies that the expression (9) is negative. Therefore, there is never an investment for $x_i<k_i^{h_{0,1}^{\tau}}$ and hence $k_i^{h_{0,0}^{\tau-1}}\geq k_i^{h_{0,1}^{\tau}}$.

These two observations imply that $k_i^{h_{0,0}^{\tau}}\geq k_i^{h_{0,0}^{\tau-1}}\geq k_i^{h_{0,1}^{\tau}}$ for $i=1,2$.

Now suppose that $\tau\geq 3$ and let us analyze the optimal behavior in period $\tau-2$. Before we proceed note that the period τ might have any relevance in determining the optimal behavior in period $\tau-2$ only after history $h_{0,0}^{\tau-2}$. So, suppose that player i observes history $h_{0,0}^{\tau-2}$. Then, his expected payoff from investment in period $\tau-2$ is

$$\mathbf{E}\left[\Theta|x_i,\iota^{\tau-2}\right]-F_{\mathsf{X}_j}(k_j^{**}|x_i,\iota^{\tau-2})-(\tau-3)c \tag{11}$$

where $k_j^{**}=\min\left\{k_j^{h_{0,0}^{\tau-2}},k_j^{h_{1,0}^{\tau-1}}\right\}$. The comparison of (11) with (9) directly reveals that if player i does not invest after $h_{0,0}^{\tau-2}$, he does not invest after $h_{0,0}^{\tau-1}$ either. But that means the decision at time $\tau-2$ is independent of the optimal behavior at time τ. In other words, if both players play $\mathtt{W}$ in period $\tau-2$, then they never play $\mathtt{I}$ in periods $\tau-1$ and τ; only if one of them invests in period $\tau-2$ then the other either plays $\mathtt{I}$ in period $\tau-1$ or he plays $\mathtt{W}$ in both periods $\tau-1$ and τ.

Therefore, repeating the same analysis, for $i=1,2$ we get $k_i^{h_{0,0}^{t}}\geq k_i^{h_{0,0}^{t-1}}\geq k_i^{h_{0,1}^{t}}$ for all $t=1,\cdots,\tau$. □

Proof of Proposition 1. Let $\sigma:=(\sigma_i,\sigma_j)$ be a monotone strategy profile. By Lemma 3 we know that there is no investment in the equilibrium path after the second period. That is, if $x_i\geq k_i^{h_{0,0}^{1}}$ player i invests in period 1, otherwise he waits. If his signal is $k_i^{h_{0,1}^{2}}<x_i\leq k_i^{h_{0,0}^{1}}$ he invests in the second period only if he observes that player j invested in period 1; otherwise he never invests.

Therefore it is sufficient to analyze only the first two periods $k_i^{h_{0,1}^{2}}$ and $k_i^{h_{0,0}^{1}}$ as they characterize the equilibria. From here on we will write $k_i^2:=k_i^{h_{0,1}^{2}}$ and $k_i^1:=k_i^{h_{0,0}^{1}}$.

First observe that at $h = \varnothing$ player i's payoff from investment is

$$\mathbf{E}\left[\Theta|x_i\right] - F_{X_j}(k_j^2|x_i). \tag{12}$$

If player i observes $h_{0,1}^2$, he invests in period 2 if and only if

$$\mathbf{E}[\Theta|x_i, x_j \geq k_j^1] \geq c. \tag{13}$$

Let $c^* := sup_{x_i}\mathbf{E}[\Theta|x_i, x_j \geq k_j^1]$. The left hand side of the inequality is continuous and increasing in x_i. Also by Lemma 2, $\mathbf{E}[\Theta|x_i, x_j \geq k_j^1] \geq \mathbf{E}[\Theta|x_i]$ and $\mathbf{E}[\Theta|\underline{x}] = 0$ for some $\underline{x}$. Hence, for $c < c^*$, there exists a unique k_i^2 such that (13) holds at equality at $x_i = k_i^2$.

The ex ante expected payoff of waiting in period 1 is

$$\left(1 - F_{X_j}(k_j^1|x_i)\right)\mathbf{E}[\Theta - c|x_i, x_j \geq k_j^1], \tag{14}$$

since player i invests in period 2 only after observing an investment.

Player i invests in period one if the expression (12) is at least as large as the expression (14), which we can write as

$$\begin{aligned} \mathbf{E}\left[\Theta|x_i\right] &\geq F_{X_j}(k_j^2|x_i) + \left(1 - F_{X_j}(k_j^1|x_i)\right)\mathbf{E}\left[\Theta - c|x_i, x_j \geq k_j^1\right] \\ \mathbf{E}\left[\Theta|x_i, x_j < k_j^1\right] &\geq \frac{F_{X_j}(k_j^2|x_i)}{F_{X_j}(k_j^1|x_i)} - \frac{1 - F_{X_j}(k_j^1|x_i)}{F_{X_j}(k_j^1|x_i)}c, \end{aligned} \tag{15}$$

by writing $\mathbf{E}\left[\Theta|x_i\right] = F_{X_j}(k_j^1|x_i)\mathbf{E}\left[\Theta|x_i, x_j < k_j^1\right] + (1 - F_{X_j}(k_j^1|x_i, x_j \geq k_j^1))\mathbf{E}\left[\Theta|x_i, x_j \geq k_j^1\right]$. Note that the in (15), the left hand side is continuously increasing in x_i, the first term on the right hand side is less than 1 for any x_i, and the second term on the right hand side converges to zero as x_i becomes infinitely large. Moreover, $\mathbf{E}\left[\Theta|\overline{x}\right] = 1$ for some $\overline{x} = 1$. Therefore by continuity and monotonicity there exists a unique k_i^1 such that (15) holds at equality.

For $c < c^*$, the symmetric equilibrium of the game is characterized by thresholds k^1, k^2 such that

$$\mathbf{E}[\Theta|x_i = k^2, x_j \geq k^1] = c, \tag{16}$$

$$\mathbf{E}\left[\Theta|x_i = k^1, x_j < k^1\right] - F_{X_j}(k^2|x_i = k^1, x_j < k^1) = -c\frac{1 - F_{X_j}(k^1|x_i = k^1)}{F_{X_j}(k^1|x_i = k^1)}. \tag{17}$$

Now we show that the equilibrium is unique. First let us show that the symmetric equilibrium is unique. Note that by the monotonicity of (16) for a given k^1 there exists a unique k^2. Moreover, k^2 is decreasing in k^1. Therefore uniqueness is immediate since (15) holds for a unique k^1. To see that the equilibrium is unique suppose that there is an asymmetric equilibria characterized by (k_i^1, k_i^2) and (l_j^1, l_j^2). Therefore, the following equilibrium conditions hold. Without loss of generality assume that $l_j^1 > k_j^1$. Thus, $l_j^1 > k_j^1 > k_i^2 > l_j^2$, which implies

$$c = \mathbf{E}[\Theta|x_i = k_i^2, x_j \geq l_j^1] > \mathbf{E}[\Theta|x_i = k_i^2, x_j \geq k_i^1] > \mathbf{E}[\Theta|x_i = l_j^2, x_j \geq k_i^1] = c$$

where the equalities follow from the equilibrium condition (16). A contradiction.

For $c \geq c^*$ there is no x_i for which a player invests in the second period. Therefore the game is simultaneous. We omit the analysis since it is standard. □

Proof of Proposition 2. There exists an equilibrium of the simultaneous game if $\Psi(\kappa) : [0,1] \mapsto [0,1]$, defined as $\Psi(\kappa) := F_{\mathsf{X}_j}(\kappa | x_i = \kappa)$, has a fixed point. But since the random variables Θ, E_i and E_j are absolutely continuous, the result follows from Brouwer fixed point theorem.[20]

In order to establish existence and uniqueness of the limiting equilibrium, we first show that for any two sequences of equilibria $\{\kappa^n\}$ and $\{\lambda^n\}$, if $\lim_{n\to\infty} \kappa^n = \kappa$ and $\lim_{n\to\infty} \lambda^n = \lambda$ then $\kappa = \lambda$. Without loss of generality suppose that $\lambda < \kappa$. Since the random variable X_j^n conditional on $x_i^n = x$ weakly converges to x, we get $\lim_{n\to\infty} F_{\mathsf{X}_j^n}(\lambda | x_i^n = \kappa) = 0$ and $\lim_{n\to\infty} F_{\mathsf{X}_j^n}(\lambda | x_i^n = \lambda) > 0$ which contradicts $\lambda < \kappa$. Since the sequences are bounded, by the Bolzano-Weierstrass theorem there is a unique $\kappa \in (0,1)$ such that any sequence of equilibria converges to κ. ☐

Proof of Proposition 3. The random variable X_j^n conditional on $x_i^n = x$ weakly converges to x. Also we know that k^1 and k^2 are continuously and inversely related. Therefore, we get the result from (16).

☐

---

[20] There are weaker sufficient conditions to guarantee existence, yet we omit this discussion for brevity.

—not for publication—

B. EXPERIMENTAL INSTRUCTIONS

GENERAL INSTRUCTIONS

This is an experiment on the economics of decision-making. Your earnings will depend partly on your decisions, partly on the decisions of others and partly on chance. By following the instructions and making careful decisions you will earn varying amounts of money, which will be paid at the end of the experiment. Details of how you will make decisions and earn money are explained below.

In this experiment, you will participate in 40 **independent** decision problems (rounds). In all rounds, you will be **randomly** matched with another participant. In what follows, we will refer to the person with whom you are matched as your *match*. After each round, you will be **randomly** matched with another participant for the next decision problem, and so on. At no point in the experiment will you know the identity of your matches.

DECISION PROBLEM

In each round you will be asked to make a choice between two alternatives A and B. Your match will face the same choice problem. Your decision and your match's decision result in the following earnings (the explanation of Q will be given later):

- If you choose A and your match chooses A: You earn Q and your match earns Q points.
- If you choose A and your match chooses B: You earn $Q - 20$ and your match earns 25 points.
- If you choose B and your match chooses A: You earn 25 and your match earns $Q - 20$ points.
- If you choose B and your match chooses B: You earn 25 and your match earns 25 points.

The following table lists your alternatives A and B in the rows, and your match's alternatives in the columns. For example, the situation in which you play A and your match plays B corresponds to the upper right cell. The numbers in that cell indicate the payoffs. The first number is your payoff (boldfaced) and the second number following the comma is your match's payoff (italicized). For instance, in the previous example, you earn a payoff of $Q - 20$ and your match earns 25 .

	A	*B*
A	**Q**, *Q*	**Q − 20**, *25*
B	**25**, *Q − 20*	**25**, *25*

WHAT IS Q?

When you play A, your earnings depend on your match's decision and on Q. Q is a number (up-to two decimals) between 20 and 50 randomly determined by the computer. That means any number between 20 and 50 is equally likely to be picked by the computer.

The computer picks Q before each round and the numbers are independent across rounds. That is, the Q chosen by the computer in the first round does not play any role on what Q will be in other rounds.

Before you make a decision you will not be told what Q is but instead you will receive an estimate of Q, which we will denote by E. Let's be more precise. After the computer randomly determines Q, it also picks a random number (up-to two decimals) between $Q - 5$ and $Q + 5$. This is your estimate E. Any number between $Q - 5$ and $Q + 5$ is equally likely to be picked by the computer. Although E does not tell you what Q exactly is, it gives an estimate of it. For example if you receive an estimate $E = 32.73$, then you know that Q is **not less than** $32.73 - 5 = 27.73$ and it is **not more than** $32.73 + 5 = 37.73$.

Note that although Q will be the **same** for both you and your match, your **estimates** can be different. That is, for the same Q, the computer also randomly picks another estimate exactly in the same manner for your match. Your estimate and your match's estimate are chosen independently. Therefore, it is very likely that they will be different numbers; however, both estimates will be between $Q - 5$ and $Q + 5$.

YOUR DECISION

After you are given your estimate, E, you are ready to make a decision. There are 3 stages in which you can finalize your decision. Note that both Q and E are **fixed** for all three stages for both you and your match. In each stage, you can choose either A or B. Choosing A is **irreversible**, while choosing B is **reversible**. That means choosing A in any stage ends the round and your earnings for that round are determined according to the table we discussed above. However, if you choose B, in either stage 1 or stage 2, then you will be allowed to revise your choice in the following stage. Note that for each stage that you choose B, your payoff will be reduced by **2 points** in case you end up choosing A. For example, if you choose B in stages 1 and 2, and then choose A in stage 3, $4 = 2 \times 2$ points will be subtracted from your earnings. On the other hand, if you also chose B in stage 3, then no extra points will be subtracted.

In any given stage, you will not observe the decision taken by your match in that stage, but you will observe decisions from **earlier** stages. For example, consider the screen below. It is currently the second stage, and as you can see, both you and your match chose **B** in the first stage; you also see that your estimate of Q is 34.02. However, you do not see your match's choice in stage 2. Since B is reversible, both you and your match can choose between A and B in stage 2.

PAYOFFS

Your potential earnings in each round depends on your choice, on your match's choice, and on Q as well as the timing of your choices. After both you and your match have made your choices, you will see the following screen. On the left, you see your estimate of Q, the true value of Q, and your profit; while on the right, you see the choices of both you and your match made in each of the 3 stages. In this example, you see that while your estimate of Q was 34.02, its true value was 33.87. You also see that your profit was 31.87: since both you and your match eventually chose A, you received $Q = 33.87$ points, but since you chose B in stage 1, 2 point was subtracted from this total.

At the end of the 40 rounds, we will add all your earnings in order to determine your total points. This total will be converted to a dollar amount according to the rule:

$1 = 100 points

This amount will then be added to the $8.00 participation fee to give your final payment. Payments will be made in private via petty cash vouchers after the completion of the experiment.

RULES

Please do not talk with anyone during the experiment. We ask everyone to remain silent until the end of the last decision problem.

Your participation in the experiment and any information about your earnings will be kept strictly confidential. Your receipt of payment and the consent form are the only places on which your name will appear. This information will be kept confidential in the manner described in the consent form.

If you have any questions please ask them now. If not, we will proceed to the experiment.

C. COMPUTATIONS

UNIFORM PRIOR AND UNIFORM NOISE

This example assumes Θ and E_i are distributed uniformly between $a < b$ and $-e < e$, respectively. We first compute the posterior on Θ for a given signal x_i.

POSTERIOR ON Θ FOR A GIVEN SIGNAL x: The posteriors are all uniform with different supports depending on the realization of x since a given realization of x can be generated by a realization of $\theta \in \big[\max\{x-e, a\}, \min\{x+e, b\}\big]$. The posterior is therefore given by

$$f_\Theta(\theta|x) = \begin{cases} \frac{1}{x+e-a} & \text{if } x \in [a-e, a+e], \\ \frac{1}{2e} & \text{if } x \in [a+e, b-e], \\ \frac{1}{-x+e+b} & \text{if } x \in [b-e, b+e], \end{cases} \quad \text{hence, } \mathbf{E}[\Theta|x] = \begin{cases} \frac{x+e+a}{2} & \text{if } x \in [a-e, a+e], \\ x & \text{if } x \in [a+e, b-e], \\ \frac{x-e+b}{2} & \text{if } x \in [b-e, b+e]. \end{cases}$$

POSTERIOR ON X_j FOR A GIVEN x_i: There are two critical regions that we should look at.

Interior Case: $x_i \in [a+e, b-e]$.
For any given x_j, the highest possible θ $x_j + e$ whereas the lowest one is $x_j - e$. Conditional on θ, each realization x_j has probability $\frac{1}{2e}$. Applying the previous findings we get

$$\begin{aligned} f_{\mathsf{X}_j}(x_j|x_i) &= \int f_{\mathsf{X}_j}(x_j|\theta) f_\Theta(\theta|x_i) d\theta = \int_{\max\{x_i-e, x_j-e\}}^{\min\{x_i+e, x_j+e\}} \frac{1}{4e^2} d\theta, \\ &= \begin{cases} 0 & \text{if } x_j < x_i - 2e, \\ \frac{x_j - x_i + 2e}{4e^2} & \text{if } x_j < x_i, \\ \frac{x_i - x_j + 2e}{4e^2} & \text{if } x_j \geq x_i, \\ 0 & \text{if } x_j > x_i + 2e, \end{cases} \end{aligned}$$

where it can be seen that the posterior on X_j is symmetric and its support is $[x_i - 2e, x_i + 2e]$. Notice that x_j and x_i are never more than $2e$ apart so that $x_j - x_i + 2e > 0$ when $x_j < x_i$ and $x_i - x_j + 2e \geq 0$ when $x_j \geq x_i$.

Boundary Cases—Case 1: $x_i \in [a-e, a+e]$**.**
In this case, the posterior of θ for a given x_i has support $[a, x_i + e]$. The domain of the integral yielding $f_{\mathsf{X}_j}(x_j|x_i)$ is $\theta \in [a, x_j + e]$ if $x_j \in [a-e, x_i]$, $\theta \in [a, x_i + e]$ if $x_j \in (x_i, a+e]$, and $\theta \in [x_j - e, x_i + e]$ if $x_j \in (a+e, x_i + 2e]$. To summarize:

$$f_{\mathsf{X}_j}(x_j|x_i) = \begin{cases} 0 & \text{if } x_j < a - e \\ \frac{x_j + e - a}{2e(x_i + e - a)} & \text{if } x_j \in [a-e, x_i) \\ \frac{1}{2e} & \text{if } x_j \in [x_i, a+e) \\ \frac{x_i - x_j + 2e}{2e(x_i + e - a)} & \text{if } x_j \in [a+e, x_i + 2e) \\ 0 & \text{if } x_j \geq x_i + 2e \end{cases}$$

Boundary Cases—Case 2: $x_i \in [b-e, b+e]$.
Applying similar arguments as in Case 1 we get

$$f_{X_j}(x_j|x_i) = \begin{cases} 0 & \text{if } x_j < x_i - 2e \\ \frac{x_j - x_i + 2e}{2e(b - x_i + e)} & \text{if } x_j \in [x_i - 2e, b - e) \\ \frac{1}{2e} & \text{if } x_j \in [b - e, x_i) \\ \frac{b - x_j + e}{2e(b - x_i + e)} & \text{if } x_j \in [x_i, b + e] \\ 0 & \text{if } x_j > b + e \end{cases}$$

Now we can compute $F_{X_j}(k|x_i)$. We restrict attention to the interior case of $x_i \in [a+e, b-e]$. There are two cases:

1. $k \in [x_i - 2e, x_i]$. In this case:

$$F_{X_j}(k|x_i) = \int_{x_i - 2e}^{k} \frac{x_j - x_i + 2e}{4e^2} dx_j = \frac{(k - x_i + 2e)^2}{8e^2}$$

2. $k \in (x_i, x_i + 2e]$. In this case:

$$F_{X_j}(k|x_i) = 1 - \int_{k}^{x_i + 2e} \frac{x_i - x_j + 2e}{4e^2} dx_j = 1 - \frac{(x_i - k + 2e)^2}{8e^2}$$

Hence

$$F_{X_j}(k|x_i) = \begin{cases} 0 & \text{if } k < x_i - 2e \\ \frac{(k - x_i + 2e)^2}{8e^2} & \text{if } k \in [x_i - 2e, x_i] \\ 1 - \frac{(x_i - k + 2e)^2}{8e^2} & \text{if } k \in [x_i, x_i + 2e] \\ 1 & \text{if } k > x_i + 2e \end{cases}$$

Posterior on Θ after observing $x_j \leq k$ or $x_j > k$: Let us start with $f_\Theta(\theta|x_j \leq k, x_i)$.

$$\begin{aligned} f_\Theta(\theta|x_j \leq k, x_i) &= \frac{F_{X_j}(k|\theta) f_\Theta(\theta|x_i)}{F_{X_j}(k|x_i)} \\ &= \begin{cases} \frac{\mathbf{1}_{[x_i - e, x_i + e] \cap [a, k - e]} \frac{1}{2e}}{F_{X_j}(k|x_i)} & \text{if } \theta \in [a, k - e] \\ \frac{\mathbf{1}_{[x_i - e, x_i + e] \cap [k - e, k + e]} \frac{1}{4e^2} \left(\int_{\theta - e}^{k} d\epsilon \right)}{F_{X_j}(k|x_i)} & \text{if } \theta \in [k - e, k + e] \\ 0 & \text{if } \theta \in [k + e, b] \end{cases} \end{aligned}$$

Again, there are two cases: $k \leq x_i$ and $k > x_i$ (with $|k - x_i| \leq 2e$).

1. $k \leq x_i$. In this case the posterior is

$$f_\Theta(\theta|x_j \leq k, x_i) = \frac{\mathbf{1}_{[x_i - e, k + e]} \frac{1}{4e^2} \left(\int_{\theta - e}^{k} d\epsilon \right)}{F_{X_j}(k|x_i)}$$

2. $k > x_i$. In this case the posterior is

$$f_\Theta(\theta|x_j \leq k, x_i) = \frac{\mathbf{1}_{[x_i-e,k-e]}\frac{1}{2e}}{F_{X_j}(k|x_i)} + \frac{\mathbf{1}_{[k-e,x_i+e]}\frac{1}{4e^2}\left(\int_{\theta-e}^{k} d\epsilon\right)}{F_{X_j}(k|x_i)}$$

The posterior $f_\Theta(\theta|x_j > k, x_i)$ is given by:

$$\begin{aligned} f_\Theta(\theta|x_j > k, x_i) &= \frac{\big(1 - F_{X_j}(k|\theta)\big) f_\Theta(\theta|x_i)}{1 - F_{X_j}(k|x_i)} & (18) \\ &= \begin{cases} 0 & \text{if } \theta \in [a, k-e] \\ \frac{\mathbf{1}_{[x_i-e,x_i+e]\cap[k-e,k+e]}\frac{1}{4e^2}\left(\int_{k}^{\theta+e} d\epsilon\right)}{1-F_{X_j}(k|x_i)} & \text{if } \theta \in [k-e, k+e] \\ \frac{\mathbf{1}_{[x_i-e,x_i+e]\cap[k+e,b]}\frac{1}{2e}}{1-F_{X_j}(k|x_i)} & \text{if } \theta \in [k+e, b] \end{cases} & (19) \end{aligned}$$

There are the usual two cases: $k \leq x_i$ and $k > x_i$ (with $|k - x_i| \leq 2e$).

1. $k \leq x_i$. In this case the posterior is

$$f_\theta(\theta|x_j > k, x_i) = \frac{\mathbf{1}_{[x_i-e,k+e]}\frac{1}{4e^2}\left(\int_{k}^{\theta+e} d\epsilon\right)}{1 - F_{X_j}(k|x_i)} + \frac{\int_{k+e}^{x_i+e}\frac{1}{2e}}{1 - F_{X_j}(k|x_i)}$$

2. $k > x_i$. In this case the posterior is

$$f_\theta(\theta|x_j > k, x_i) = \frac{\mathbf{1}_{[k-e,x_i+e]}\frac{1}{4e^2}\left(\int_{k}^{\theta+e} d\epsilon\right)}{1 - F_{X_j}(k|x_i)}$$

It can be verified that the posterior in the last expression and the posterior for the case in which $x_i \leq a + e$ and $k > a + e$ are identical.

EQUILIBRIUM CONDITIONS

We restrict ourselves to the interior case and then verify that equilibrium thresholds cannot belong to either boundary case.

SEQ. **Second period condition:** If $k^1 > k^2$, substituting from the previously derived expression we have:[21]

$$\begin{aligned}
\mathbf{E}[\Theta|x_j > k^1, k^2] &= \frac{\int_{k^1-e}^{k^2+e} \theta \frac{1}{4e^2}\left(\int_{k^1}^{\theta+e} d\epsilon\right) d\theta}{1 - F_{X_j}(k^1|k^2)} = \frac{\int_{k^1-e}^{k^2+e} \frac{\theta(\theta+e-k^1)}{4e^2} d\theta}{\frac{(k^2-k^1+2e)^2}{8e^2}} \\
&= 2\frac{\int_{k^1-e}^{k^2+e} \theta^2 d\theta - (k^1-e)\int_{k^1-e}^{k^2+e} \theta d\theta}{(k^2-k^1+2e)^2} \\
&= 2\frac{\frac{(k^2-k^1+2e)^2}{12} + \left(\frac{k^2+k^1}{2}\right)^2 - (k^1-e)\frac{k^2+k^1}{2}}{(k^2-k^1+2e)} \\
&= 2\frac{\frac{(k^2-k^1+2e)^2}{12} + \frac{k^1+k^2}{4}(k^2-k^1+2e)}{(k^2-k^1+2e)} \\
&= \frac{k^2-k^1+2e}{6} + \frac{k^1+k^2}{2} \\
&= \frac{2k^2+k^1+e}{3}
\end{aligned}$$

In the second period a player invests conditional on observing one investment if $\mathbf{E}[\theta|x_j > k^1, x_i] > c+s$ where c is the waiting cost and s is the payoff from the safe action. Therefore, for a given k_1, $k^2 < k^1$ and $k^2 - k^1 > -2e$, the second-period threshold is given by:

$$k^2 = \frac{3}{2}(c+s) - \frac{e}{2} - \frac{k^1}{2}$$

First period condition with $k^1 > k^2$: for a given loss from investing alone L, waiting cost c and safe action payoff s, from the expressions previously derived we have:

$$\begin{aligned}
\mathbf{E}[\Theta|x_j \leq k^1, k^1] - LF_{X_j}(k^2|x_j \leq k^1, k^1) &= \frac{1}{e}\int_{k^1-e}^{k^1+e} \theta \frac{k^1-\theta+e}{2e} d\theta - L\frac{(k^2-k^1+2e)^2}{4e^2} \\
&= \frac{(k^1+e)k^1 - L\frac{4e^2}{12} - (k^1)^2}{e} - \frac{(k^2-k^1+2e)^2}{4e^2} \\
&= k^1 - \frac{e}{3} - L\frac{(k^2-k^1+2e)^2}{4e^2}
\end{aligned}$$

Therefore, the equilibrium threshold k^1 is given by:

$$k^1 - \frac{e}{3} - L\frac{(k^2-k^1+2e)^2}{4e^2} + c - s = 0,$$

---

[21] Notice that $\int_{k^1-e}^{k^2+e} \theta d\theta$ is the expected value of a uniformly distributed random variable multiplied by the total mass. That is, $\int_{k^1-e}^{k^2+e} \theta d\theta = (k^2-k^1+2e)\frac{k^2+k^1}{2}$. By a similar argument, $\int_{k^1-e}^{k^2+e} \theta^2 d\theta = (k^2-k^1+2e)\left[\frac{(k^2-k^1+2e)^2}{12} + \left(\frac{k^2+k^1}{2}\right)^2\right]$.

and substituting the expression for k^2, we obtain:

$$k^1 - \frac{e}{3} - L\left(\frac{3}{4e}\right)^2 (c+s+e-k^1)^2 + c - s = 0.$$

SNC. **Equilibrium conditions:** The second period condition with $l^2 < l^1$ satisfies:

$$l^2 = \frac{3}{2}(c+s) - \frac{e}{2} - \frac{l^1}{2}$$

The first period condition satisfies:

$$l^1 - \frac{e}{3} + c - s = 0$$

and solving for l^2 yields:

$$l^2 = 2c + s - \frac{2}{3}e$$

It is easy to verify that $l^2 < l^1$ if $e > 3c$ and that the same condition implies that $l^1 > s$ so that both thresholds are well defined.

In the experiments where $e \leq 5$ and $c \leq 4$, it can be readily verified that $e \leq 3c$ and that the above equilibrium conditions do not apply. This is because the waiting cost is too high relative to the amount of information that can be gathered by delaying.

In this case, the equilibrium is to invest if one's signal is above s. Clearly, a player does not invest in the first period if $E[\Theta|x_i] \leq s$ or $x_i \leq s$. Suppose that the first-period equilibrium threshold is $l^1 > s$. Then it must be the case that for $s \leq x_i \leq l^1$ it is optimal to delay. For given l^1, we have that:

$$E[\Theta|x_j > l^1, x_i] - (c+s) = \frac{2x_i + l^1 + e}{3} - (c+s)$$

which is negative for all $x_i \leq l^1 \leq s + c - \frac{e}{3}$. This means that player i invests the first period for $s \leq x_i \leq l^1 \leq s + c - \frac{e}{3}$ and $l^1 \in (s, s + c - \frac{e}{3}]$ cannot be an equilibrium. Now take $l^1 > s + c - \frac{e}{3}$. If this were an equilibrium, then we would have

$$E[\Theta|l^1] - s = \left[E[\Theta|x_j > l^1, l^1] - (c+s)\right][1 - F_{X_j}(l^1|l^1)] = [l^1 + \frac{e}{3} - c - s]\frac{1}{2}$$

but the left-hand-side equals $l^1 - s$ and the first argument of the right-hand-side is positive but cannot be higher than $l^1 - s$ because $\frac{e}{3} - c < 0$.

Finally, for $x_i < l^1 = s$ the payoff from delaying and investing in the second period is negative. Therefore, the equilibrium requires that players invest in the first period is $x_i > s$ or otherwise delay indefinitely.