Acquisition of Information to Diversify Contractual Risk*

Boğaçhan Çelen†　　Saltuk Özertürk‡

Columbia University　　SMU

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Abstract

This paper analyzes a principal-agent problem whereby the agent can trade in financial markets in order to diversify his compensation risk. Prior to making a portfolio decision, the agent acquires information on how the financial assets available in the market will fit his diversification purposes. We model this information-acquisition activity as a costly search process. The amount of risk that the agent diversifies decreases in information-acquisition cost and increasing in the financial market's sophistication, as measured by the variety of financial assets available. As the agent gains access to a financial market with lower information acquisition costs and higher sophistication, the optimal compensation contract involves more performance-sensitive pay and elicits a higher level of effort.

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†Graduate School of Business, Columbia University, 3022 Broadway, 602 Uris Hall, New York, NY 10027. E-mail: bc2132@columbia.edu, url: http://celen.gsb.columbia.edu/.

‡Department of Economics, Southern Methodist University, Dallas, TX 75275. E-mail: ozerturk@mail.smu.edu, url: http://faculty.smu.edu/ozerturk/.

1 INTRODUCTION

The trade-off between efficient risk sharing and provision of incentives is at the heart of the principal-agent framework. The optimal contract trades the benefits of eliciting more effort from an agent by making compensation more sensitive to performance for the insurance consideration attributable to agent's inability to diversify.[1] Almost axiomatically, most treatments of the principal-agent model assume that, while principal can diversify any exposure to contractual risk by holding a well- diversified portfolio of assets, agent has no access to similar portfolio opportunities. Although this assumption provides a convenient benchmark, there are many economic environments in which agents can engage in private financial transactions and hold asset portfolios.[2] Optimal portfolio theory informs us that risk-averse agents will exploit opportunities to lower the risks in their compensation contracts.[3]

Agency theory, on the other hand—while ignoring the agent's diversification incentive for the most part[4]—is somewhat silent as to exactly what undermines the agent's diversification ability. To be sure, certain types of risks—like the market value of the agent's human capital—are extremely difficult, if not impossible, to diversify, even with sophisticated financial instruments. However, given the recent advances in financial innovations, agents do seem to have access to portfolio opportunities that can reduce their contractual risks.

In this paper, we consider an extension of the standard principal-agent model that allows the agent to trade portfolios correlated with his contractual risk. While our agent has access to financial markets, this access is subject to imperfections that we model explicitly. One feature of our approach is informational frictions: the agent does not have full information about how all of the assets in the market fit his diversification purposes, and hence needs to engage in costly acquisition of information before making a portfolio decision. We model this information-acquisition activity as a search process. After the compensation contract is set, but before choosing effort, the agent seeks a portfolio to diversify his compensation risk. For the agent's diversification purposes, available portfolios differ in their fit. This fit is measured by the correlation of a portfolio's payoff

[1]See Holmström (1979), Laffont and Martimort (2002, Chapter 4) and Bolton and Dewatripont (2005, Chapter 4) for the standard analysis of the trade-off between insurance and incentives.

[2]A recent example is the emergence of a sizable hedge market for corporate executives. Bettis et al. (2001) report that in the late 1990s investment banks developed and introduced sophisticated financial instruments enabling corporate executives to diversify their stock ownership positions.

[3]This diversification incentive is well documented in the empirical corporate finance literature that links corporate risk management policies to managerial diversification motives (Amihud and Lev (1981), May (1995), and Tufano (1996)). These papers present empirical evidence supporting the idea that managers have an incentive to implement inefficiently low risk investment policies in order to lower the risks in their compensation schemes. We briefly discuss related issues in corporate risk management literature in Section 4.

[4]We will discuss the exceptions in the agency literature shortly.

with the agent's idiosyncratic risk. Ex ante, the agent only knows the distribution of correlations, but not the correlation of a particular portfolio. By costly sampling from the set of available portfolios, the agent can learn the correlation of a portfolio. If the agent is content with how well a given portfolio fits his diversification purposes, he stops sampling and chooses a position in that portfolio. If not, he continues sampling.

This search problem endogenizes the amount of compensation risk that the agent diversifies. The agent searches more aggressively in the financial market and diversifies more risk: (*1*) as the cost of information acquisition in the market (measured by the search cost) decreases; and (*2*) as the sophistication of the financial market (measured by the variety of financial portfolios available) increases. One novelty of our model is the introduction of these two financial market characteristics into the principal-agent environment as determinants of the optimal pay-performance sensitivity and contracting efficiency. We show that, as the agent gains access to a financial market with lower information-acquisition costs and higher sophistication, the optimal compensation contract involves more performance-sensitive pay and elicits a higher level of effort. This result follows, because as the agent can diversify more risk, the incentive consideration becomes more important than the insurance consideration in determining the optimal compensation scheme, resulting in higher pay-performance sensitivity and effort.

This positive result on incentives is especially interesting in the context of the recent debate on the efficiency implications of hedge markets for corporate executives. In the late 1990s, financial firms in the U.S. developed and introduced sophisticated trading opportunities enabling the corporate managers to hedge their stock ownership positions (cf. Bettis et al. (2001).) The general view on the availability of such trading opportunities, mostly shaped by the business press and scholarship in the legal profession, has been quite negative. It is typically argued that if the managers have unrestricted access to financial markets, they will trade and hedge the performance incentives in their compensation schemes, rendering the incentive justification for managerial stock ownership invalid (Bank (1995), and Easterbrook (2002).) We contribute to this debate by describing a hedge transaction that actually improves (rather than undermines) incentive contracting.

Our analysis illustrates that the availability of financial portfolios correlated with firm specific risk can achieve a more efficient contracting outcome, as they allow to reduce the randomness in the risk averse agent's compensation scheme. We should note that since such trades increase the contracting efficiency in a principal-agent setting, they are perfectly desirable also from the point of view of the principal: the more risk the agent can diversify by his trades, the better off is the principal. This observation raises a related question: can the principal also use such correlated portfolios in the initial contract himself, and provide the agent with a less risky initial contract, rather than having the agent search and customize his own hedge portfolio?

We address this question in an extension where the principal can also engage in costly search for financial portfolios to include in the agent's initial compensation scheme. In other words, we allow the principal to tie the agent's compensation not only on firm value, but also on financial portfolios that are correlated with firm specific risk. After receiving this initial contract, the agent can still search for improved hedging opportunities if he desires to do so. In that setting, we show that whether the agent or the principal undertakes the costly portfolio search depends crucially on the relative search costs of the parties. Since the agent's and the principal's incentives are completely aligned in terms of how much risk to diversify from the agent's compensation scheme, the search is optimally undertaken by the party with lower search costs.

In particular, there is no equilibrium where both the principal and the agent search for financial portfolios. We show that for the same level of diversification, when the search costs are identical, it is always optimal for the principal to do the search and provide diversification in the contract. Letting the agent do the search is optimal, if the agent's search cost is relatively low enough compared to the principal's search cost. In that respect, the main insight of this paper is to illustrate that there may be efficiency benefits associated with using financial markets to hedge compensation risks: since this efficiency benefit increases welfare of both the principal and the agent, the precise optimal implementation of this hedging (whether the agent or the shareholders do the hedging) depends on which party has lower cost of unveiling these portfolio opportunities.

RELATED LITERATURE The papers that consider agency settings in which the agent can trade in financial markets do so in models where the agent can either privately borrow and save (i.e., trade debt contracts), or trade side contracts contingent on his own output. These studies typically find that the agent's trading ability undermines contracting efficiency and incentives. In a two-period principal-agent model, Rogerson (1985) shows that if the agent has no access to riskless saving and borrowing the optimal contract will leave the agent with a precautionary demand for saving. Riskless saving benefits a risk-averse agent by providing partial insurance against future wage uncertainty, but this insurance weakens incentives and lowers the equilibrium effort that is optimally elicited.[5] Park (2004) allows the agent to privately borrow and save in the *pre- contracting stage*, and shows that the agent's pre-contracting access to private credit markets results in a severe loss of incentive provision.[6] Garvey (1993) and Bisin et al. (2008) allow the

[5]Bizer and DeMarzo (1999) allow for risky borrowing and saving. They show that the efficiency implications of the agent's access to credit markets depend crucially on the treatment of default.

[6]In Park's paper, the agent's access to private credit markets before the principal offers a contract creates an endogenous informational asymmetry regarding the contractual relationship. The principal does not observe the agent's pre-contracting savings, hence he also does not know the agent's preferences as to the subsequent contracts.

agent to trade side contracts contingent on his own output *after* the compensation contract is set.[7] They find that the side trades contingent on own output also undermine incentives. In a similar vein, our earlier paper Çelen and Özertürk (2007) shows that a manager's ability to trade non-exclusive swap contracts (promising the return from his shares to third parties in exchange for a fixed payment) can undo the link between firm performance and compensation: as such, swap contracts can undermine incentives completely.

In this paper, we show how the agent can manage to reduce the randomness in his compensation scheme by customizing a portfolio correlated with his firm specific risk. The agent's ability to trade a portfolio correlated with his contractual risk improves, rather than undermines, incentives. The crucial reason for this result is that, unlike borrowing and saving, or side contracts contingent on the agent's own output, a portfolio correlated with the agent's idiosyncratic risk serves to reduce the randomness in his wealth but preserves the link between his wealth distribution and his subsequent choice of effort. Since the agent can lower the contractual risk by holding an optimally chosen position in a correlated portfolio, he demands a lower risk premium for a given exposure to the stochastic output. As a result, the insurance cost of eliciting a given effort level decreases, improving contracting efficiency.

In contrast, the agent's side trades contingent on own output (as modeled in Garvey (1993) and Bisin et al. (2008)) serve to undo the link between his effort choice and wealth. With such side trades, the agent simply promises his share of output to third parties and unwinds the effort incentives provided by the contract. In other words, while the agent's trades in portfolios correlated with his risk reduce the noise in his wealth distribution for a given effort choice (lowering the insurance cost of incentive provision), the side contracts contingent on own output reduce the link between a given effort choice and wealth (undermining incentive provision).[8]

Our paper is also related to the literature on the information-acquisition incentives of agents before they make portfolio decisions (see for example, Grossman and Stiglitz (1980), Admati and Pfleiderer (1986), and Peress (2004).)[9] In contrast to these models of

[7]In Bisin et al. (2008), the principal can engage in costly and imperfect monitoring of the agent's portfolio to prevent the agent from side trades contingent on own output.

[8]The literature allowing the agent to make non-exclusive contracts with multiple principals for the same moral hazard activity also typically reports that the agent's unobservable side trades undermine contracting efficiency (see, for example, Kahn and Mookherjee (1995, 1998) and Bisin and Guaitoli (2004) on the implications for insurance and credit markets, and Parlour and Rajan (2001) for loan contracts from multiple creditors.) Cole and Kocherlakota (2000) consider an environment where agents can privately store assets for insurance purposes, and provide a characterization of the efficient consumption allocation.

[9]These papers adopt rational-expectations frameworks in which the agents also can learn from asset prices that aggregate market information. They relate the agent's information-acquisition incentives to risk aversion (which determines the intensity that the agent trades on private information); to the asset supply noise (which determines the informativeness of asset prices); and, in the case of Peress (2004), to the agent's wealth level.

information acquisition in asset markets, our model has an agent who trades only for diversification purposes and is concerned solely with learning how different portfolios are correlated with his contractual risk: his incentives to acquire information and to trade in the financial market arise endogenously in response to the risk in his compensation scheme. Since the information that our agent seeks is agent-specific, the information-acquisition activity that we model as a search process is quite different from that of existing models where the agents receive private signals about asset returns. As in this paper, Gale (1997) adopts a search framework in a setting where investors are uncertain about the qualities of different financial assets and engage in costly information-acquisition activity. However, Gale's paper focuses on the inefficiencies that arise when the agents have too many assets to choose from, because they engage in excessive search.

The paper proceeds as follows. The next section lays out the model. Section 3 provides the analysis and contains our results. Section 4 provides a discussion and Section 5 concludes. We relegate cumbersome proofs to an Appendix.

2 The Model

The basic model is based on the commonly used CARA-normal principal-agent model with linear contracts. It is detailed below.

Technology and Preferences. A risk-neutral principal hires a risk-averse agent. The agent chooses unobservable effort, $e \in [\underline{e}, \bar{e}] \subset \mathbb{R}$, which, together with the realization of a random shock ϵ, determines the final output x:

$$x := f(e) + \epsilon.$$

The function $f : [\underline{e}, \bar{e}] \mapsto \mathbb{R}_+$ is non-decreasing, concave and differentiable, and represents the productivity of effort. We assume that the idiosyncratic output shock is a random variable $\mathbf{E}$, normally distributed with mean 0 and variance σ_ϵ^2. Therefore, given e, output is a random variable $\mathbf{X}_e$ induced by $\mathbf{E}$, normally distributed with mean $f(e)$ and variance σ_ϵ^2. Supplying effort is costly for the agent. The cost of effort is determined by a non-decreasing, convex, differentiable function

$$c : [\underline{e}, \bar{e}] \mapsto \mathbb{R}_+.$$

The risk-averse agent has constant absolute risk aversion (CARA) preferences represented by the utility function $u : \mathbb{R}_+ \mapsto \mathbb{R}$ specified as

$$u(w) := -\exp\{-aw\}.$$

The parameter $a > 0$ is the agent's coefficient of absolute risk aversion, and w is his final wealth.

LINEAR COMPENSATION. The principal's objective is to maximize the expected output net of the agent's compensation. Since the agent's effort choice is unobservable, the principal cannot compensate the agent contingent on that effort. However, the principal can provide effort incentives by tying the agent's compensation to the realized output. Specifically, we assume that the agent's compensation scheme takes the linear form $sx + t$, where t is a fixed payment and s is the agent's share of the final output x. In what follows, we denote such a contract by (s, t), and we refer to s as the *pay-performance sensitivity* of the agent's compensation scheme.[10]

THE AGENT'S SEARCH FOR PORTFOLIOS. We assume that there is a financial market that contains portfolios that are correlated with firm specific risk $\mathbf{E}$. Although, we devote the main body of the paper to study the case in which only the agent can trade in a financial market, in Section 3.4 we also analyze an extension where the principal can also trade such portfolios. We assume that the agent's access to financial markets is subject to imperfections. The key aspect of our approach is that the agent seeks to diversify a risk that is specific to his contracting scheme, and hence needs to know how different portfolios are correlated with this particular risk.[11] It seems quite unlikely that this kind of agent-specific information is freely available in the market when the agent begins to trade. Therefore, we assume that the agent engages in costly information acquisition about how different portfolio returns are correlated with his contractual risk. By doing so, the agent learns how well a particular portfolio fits his diversification purposes. The following search framework proves to be a plausible metaphor and a tractable analytical device for analyzing the agent's information-acquisition problem. There is a variety of financial portfolios that the agent can potentially hold. The payoff from a generic portfolio is represented by a random variable $\mathbf{Y}$, which is assumed to be distributed

[10]Linear contracts have been highly attractive in the literature due to their tractability and the intuitive solution they deliver. Holmström and Milgrom (1987) show that linear contracts are optimal in a dynamic principal- agent setting with CARA preferences and with a binomial output process; their result has provided the primary justification for restricting attention to linear contracts in a variety of applications. As many other papers in the literature, our objective in using linear contracts is to have the ability to talk about the intensity of incentives, measured by s. For general treatments of the principal-agent problem, we refer the reader to Ross (1973), Harris and Raviv (1979), Holmström (1979), and Grossman and Hart (1983).

[11]Therefore, an aggregate financial market index, which helps to diversify risks common to all agents in the economy is not of much use for our agent's diversification problem. For models where the agent can trade market indices to diversify the systematic risk, see Garvey and Milbourn (2003), Jin (2002), Acharya and Bisin (2005), and Özertürk (2006). We further discuss this issue in Section 4.

normally with mean μ and variance σ_y^2.[12] Each $\mathbf{Y}$ is correlated with the output shock $\mathbf{E}$ according to $\mathsf{cov}(\mathbf{Y}, \mathbf{E})$; therefore, from the view point of the agent, $\mathbf{Y}$ serves as a potential instrument for diversifying $\mathbf{E}$. However, the degree of diversification that a portfolio provides will depend on the correlation of its payoff with $\mathbf{E}$. The agent clearly prefers a portfolio that is highly correlated with $\mathbf{E}$—either positively or negatively— and with a low standard deviation σ_y. To capture this, we define

$$q := \frac{\mathsf{cov}(\mathbf{Y}, \mathbf{E})}{\sigma_y}.$$

We refer to $q^2 =: z$ as the quality of a portfolio: a portfolio with a higher z serves the agent's diversification purposes better. Should the agent have perfect knowledge of all available portfolios, he would choose the one with the highest z in order to diversify as much risk as possible. However, our agent is not perfectly familiar with the specifics of each and every portfolio available in the market. Each available portfolio has a different quality, and the agent only knows the distribution of qualities, not the quality of a particular portfolio.

The set of financial portfolios can be described as the collection of all q's available. The agent searches for a portfolio from the distribution of all q's. More precisely, there is a random variable $\mathbf{Q}$ that describes the quality of all available portfolios. The agent samples a portfolio from $\mathbf{Q}$ and finds out its quality; that is, he learns its correlation with $\mathbf{E}$. If he is "satisfied" with the quality of the portfolio, then stops and creates a position $d \in \mathbb{R}$ in that portfolio; otherwise, he pays a cost $\kappa > 0$ and samples another portfolio, and so on, until he decides to stop the search process. If the agent creates a position $d \in \mathbb{R}$ in the portfolio at which he stopped, then he is entitled to a claim $d\mathbf{Y}$. In exchange, the agent pays the party who is trading with him pd, where $p \in \mathbb{R}_+$ is the share price of the portfolio $\mathbf{Y}$. We assume that the parties that can potentially trade with the agent are risk-neutral and competitive, so that each share of the portfolio $\mathbf{Y}$ is priced at $p = \mathsf{E}[\mathbf{Y}] = \mu$.

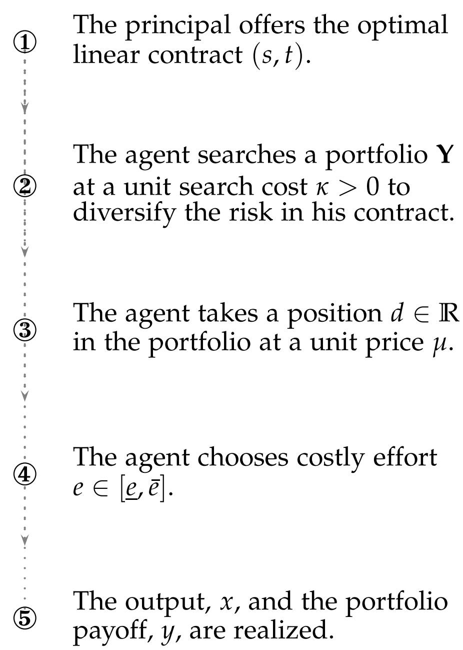
This risk-neutral pricing assumption ensures that there is no risk premium for holding a portfolio: the only incentive for the risk-averse agent to hold a portfolio is to diversify the contractual risk. Accordingly, what matters for the agent's portfolio decision is not the portfolio's expected return but the correlation structure of the portfolio return. For that reason, the assumption that all portfolios have the same expected return is irrelevant under risk-neutral pricing.

For convenience, we summarize the sequential form of the game and the notation: First, the principal offers a contract (s, t) to the agent. If the agent accepts the contract,

[12] As we argue shortly, the assumption that all available assets have the same expected return, μ, does not affect the analysis.

he searches a portfolio $\mathbf{Y}$ at a unit cost $\kappa > 0$. Following the search, the agent takes a position $d \in \mathbb{R}$ in the portfolio at which he stopped. Then, the agent chooses his effort level, e. Finally, the output x, and the payoff of the portfolio are realized. We are interested in the subgame perfect equilibrium of this game. Figure 1 depicts the timing and the notation of the game.

Figure 1: Timing of The Game



3 Analysis

3.1 A Single Correlated Portfolio

To illustrate how the agent's access to a financial portfolio correlated with his firm-specific risk improves contracting efficiency, we begin our analysis by focusing on the contracting problem when only one such portfolio is available. Furthermore, we assume that *the agent perfectly knows the quality of this portfolio*.

Suppose there is only one portfolio $\mathbf{Y}$ with a given know quality q. Given a compensation contract (s, t), if the agent chooses a position $d \in \mathbb{R}$ in that portfolio and expends an effort level e, his wealth can be written as a random variable

$$\mathbf{W}_{e,d} = s\mathbf{X}_e + d\mathbf{Y} + t - c(e) - \mu d.$$

The CARA preferences and the normality assumptions on $\mathbf{E}$ and $\mathbf{Y}$ imply that the agent's problem of choosing e to maximize $\mathsf{E}\big(u(\mathbf{W}_{e,d})\big)$ is equivalent to maximizing the *certainty equivalent wealth*[13]

$$\begin{aligned} w(e,d) &:= \mathsf{E}(\mathbf{W}_{e,d}) - \frac{a}{2}\mathsf{Var}(\mathbf{W}_{e,d}) \\ &= sf(e) + t - c(e) - \frac{a}{2}\left(s^2\sigma_\epsilon^2 + d^2\sigma_y^2 + 2sdq\sigma_y\right). \end{aligned}$$

Notice that $w(e,d)$ is concave, differentiable and non-decreasing in $e \in [\underline{e}, \bar{e}]$. Furthermore, the choice of effort only affects the expected wealth, whereas the choice of a position d only affects the variance of the agent's wealth distribution. The optimal effort e^* satisfies one of the following two conditions:

$$\begin{aligned} e^* \in (\underline{e}, \bar{e}) \text{ and } sf'(e^*) - c'(e^*) = 0, \\ e^* = \bar{e} \text{ and } sf'(e^*) - c'(e^*) \geq 0. \end{aligned} \tag{1}$$

The above expression for the optimal effort choice is standard: as the agent is given a higher pay-performance sensitivity s, he expends more effort.

Turning to the optimal choice of a position in portfolio $\mathbf{Y}$, notice that $w(e^*, d)$ is also concave, differentiable and non-monotonic in d. Therefore, straightforward maximization yields the optimal position in $\mathbf{Y}$ as

$$d^*(s) = -\frac{q}{\sigma_y}s. \tag{2}$$

The agent's optimal position in $\mathbf{Y}$ is increasing in the pay-performance sensitivity of his compensation scheme (that determines his pre-trading exposure to the idiosyncratic risk $\mathbf{E}$) and the quality q of the portfolio (that describes the correlation between $\mathbf{E}$ and the portfolio return $\mathbf{Y}$). The sign of q determines whether the optimal position involves buying or short-selling the portfolio. Furthermore, because of the risk-neutral pricing, the agent's portfolio demand arises solely because of the diversification incentive: if the risk-averse agent had no exposure to $\mathbf{E}$ through his compensation contract ($s = 0$), or if the portfolio in question provided no diversification ($q = 0$), then the agent would not hold a portfolio position.

By using (1) and (2), we can illustrate how a position in $\mathbf{Y}$ provides diversification for $\mathbf{E}$. At the optimal e^* and d^*, the agent's certainty-equivalent wealth becomes

$$w(e^*, d^*) = sf(e^*) + t - c(e^*) - \frac{a}{2}s^2(\sigma_\epsilon^2 - z).$$

By holding a position d^* in a portfolio with quality q, the agent reduces the variance of

[13]To see this, it is enough to check the joint moment-generating function of $\mathbf{X}_e$ and $\mathbf{Y}$.

his wealth distribution to $s^2(\sigma_\epsilon^2 - z)$. The ability to trade lowers the agent's disutility from his exposure to firm-specific risk, and hence lowers the associated risk premium that the he demands for bearing this risk. From the principal's perspective, this diversification ability increases contracting efficiency, since it decreases the insurance cost of eliciting a given effort level. Accordingly, the agent's equilibrium pay-performance sensitivity s, and hence the equilibrium effort that is optimally elicited should be increasing in the amount of risk that the agent can diversify by using a portfolio $\mathbf{Y}$. We verify this intuition next.

Optimal Pay-Performance Sensitivity. To describe explicitly the incentive implication of the agent's access to a portfolio $\mathbf{Y}$ correlated with $\mathbf{E}$, let us write down the principal's problem of setting the optimal compensation scheme (s, t). The principal chooses (s, t) to maximize the expected firm value net of the agent's compensation, which is given by

$$(1 - s)\mathsf{E}(\mathbf{X}_{e^*}) - t,$$

subject to agent's participation constraint $u\big(w(e^*, d^*)\big) \geq u(\bar{w})$, where $\bar{w}$ is the agent's reservation certainty equivalent wealth. We can write this participation constraint in terms of certainty equivalent wealth as

$$sf\big(e^*\big) + t - \frac{a}{2}s^2(\sigma_\epsilon^2 - z) \geq \bar{w}.$$

At the optimal contract, this participation constraint holds as an equality. Solving for t and substituting it into the principal's objective function, one can describe the problem as choosing s to maximize the net expected surplus

$$f\big(e^*\big) - c\big(e^*\big) - \frac{a}{2}s^2\big(\sigma_\epsilon^2 - z\big) - \bar{w}.$$

Since the agent's trade in a portfolio $\mathbf{Y}$ reduces the risk premium that must be paid to the agent's for bearing firm-specific risk, the expected net surplus in the contractual relationship increases in the quality z of the portfolio. The optimal pay-performance sensitivity s^* that maximizes the above net expected surplus satisfies one of the following two conditions:

$$s^* \in (0, \bar{s}) \text{ and } (1 - s)f'(e^*)\frac{de^*}{ds} = as(\sigma_\epsilon^2 - z),$$
$$s^* = \bar{s} \text{ and } (1 - s)f'(e^*)\frac{de^*}{ds} \geq as(\sigma_\epsilon^2 - z),$$

where $e^*(\tilde{s}) = \bar{e}$, and $\bar{s} = \min\{\tilde{s}, 1\}$. This condition equates the marginal benefit of increasing s (which works through eliciting higher effort) to the marginal cost of this incentive provision given by $as\left(\sigma_\epsilon^2 - z\right)$. This cost stems from the agent's aversion to bearing firm-specific risk. For a given s, the availability of a portfolio correlated with **E** allows the agent to reduce the risk to $\left(\sigma_\epsilon^2 - z\right)$ from σ_ϵ^2. Using the above condition, one can show that the optimal s^* and hence $e^*(s^*)$ increase in the quality of the portfolio. We summarize this result below.

PROPOSITION 1 *Suppose there is only one portfolio with a given known quality. The optimal pay-performance sensitivity that the principal sets and the equilibrium effort that they can elicit increase in the quality of the portfolio.*

This positive result provides an interesting contrast to various negative results in the literature on the incentive effects of an agent's side trades to hedge his compensation risk, such as in Garvey (1993) and Bisin et al. (2008). The reason that our hedging framework delivers a positive result is the particular way we allow the agent to hedge. The agent's incentive to engage in side trades to hedge his compensation risk stems from his aversion to firm specific risk. Previous papers have considered hedging transactions in which the agent can trade side contracts *based on his own firm value*. This type of hedging serves to undo the link between the agent's wealth and the performance of his firm which depends on the agent's effort choice. In the context of our model, if the agent could trade side contracts based on his own firm value **X**, he would effectively do so to reduce the pay-performance sensitivity of his compensation contract.[14] Such trades would reduce the agent's disutility from exposure to firm specific risk as well, but would do so by lowering s rather than diversifying **E**. The negative results in the literature thus relies on hedging transactions that work through reducing s, and hence undermine the agent's subsequent effort incentives.[15] In contrast, we describe a type of hedging transaction that diversifies the firm-specific risk **E**, while preserving the link between the agent's effort choice and his wealth distribution provided by s.

The key feature of the model that generates the positive result is that the agent trades a portfolio correlated with the idiosyncratic risk only, and the portfolio payoff is not affected by the agent's effort decision. This specification separates the effort decision from the agent's portfolio problem. As in the alternative specifications of a hidden effort type moral hazard framework, our principal can elicit a higher effort level by promising to pay the agent more in states which are more likely when the agent expends effort. For

[14]For example, one such hedging transaction based on own firm value is an equity swap contract in which the manager promises the return from his company shares to a third party in exchange for a fixed payment (see Bettis et al (2001)). By simulating the sale of the manager's shares, a swap transaction reduces manager's effective share ownership and dilutes the link between the manager's wealth and firm value. Therefore, from an agency perspective, swap transactions serve to undermine managerial incentives to maximize shareholder value.

[15]Of course, the third parties would anticipate the diminished effort incentives and price the hedging transaction conditional on the manager expending less effort subsequent to hedging (see for example Bisin et al. (2008).)

a given compensation scheme and the effort level that it elicits, the residual uncertainty $\mathbf{E}$ adds noise to agent's wealth distribution and reduces the risk averse agent's expected utility from that compensation scheme. As long as the agent can separately hold a position in a portfolio directly correlated with $\mathbf{E}$, he can improve upon the initial contract by reducing this noise and achieve a compensation scheme with less randomness. Accordingly, CARA preferences and the normality assumptions on $\mathbf{E}$ and $\mathbf{Y}$, although they provide tractability, do not seem to be essential for the result.

INITIAL CONTRACT BASED ON $\mathbf{Y}$. By reducing the insurance cost of incentive provision, the agent's trade in a portfolio correlated with firm-specific risk increases the efficiency of the contractual relationship between the agent and the principal. In that respect, we should note that the agent's trade in a correlated portfolio $\mathbf{Y}$ is also desirable for the principal. Indeed, if we assume that the principal can also trade the correlated portfolio and base the initial contract on $\mathbf{Y}$ as well as firm value $\mathbf{X}$, then *he would be able to achieve the same efficiency in terms of the optimal effort elicited*. In particular, suppose the principal could offer a contract (s, α, t)—that takes the form of wealth $s\mathbf{X}_e + \alpha\mathbf{Y} + t$—to the agent. In this case, the principal would exactly choose the same exposure to $\mathbf{Y}$ as the agent chooses with d^* above, and use $\mathbf{Y}$ to minimize the variance associated with $s\mathbf{X}_e$. Formally, the principal would always set an initial contract $s\mathbf{X}_e + \alpha\mathbf{Y} + t$ such that

$$\alpha = -\frac{q}{\sigma_y}s.$$

Since this contract would already make full use of $\mathbf{Y}$ in minimizing $\mathsf{Var}(s\mathbf{X}_e + \alpha\mathbf{Y})$, the agent would have no further demand for $\mathbf{Y}$ even if he had a further trading opportunity in $\mathbf{Y}$. As a result, if the principal could offer a contract (s, α, t) to the agent, the optimal pay-performance sensitivity and the equilibrium effort would exactly be the same as described in Proposition 1.

The analysis up to this point shows that the availability of a portfolio correlated with firm-specific risk increases contracting efficiency, and the extent of this efficiency benefit is increasing in the correlation of the portfolio available. In making this point, we assume that there is only one portfolio with a known correlation. Although this is a convenient assumption for illustrative purposes, it does not allow one to be more specific about what determines the amount of diversification that can be achieved. In practice, what kind of frictions does the agent face while customizing his hedge portfolio? One could exogenously specify a "cost of diversification" function for the agent, and establish some comparative statics results on contracting efficiency with respect to this exogenous cost function. Yet, this reduced form approach does not seem to be satisfactory in terms of explaining why it is costly for the agent to find out and trade the best portfolio in the first place. Similarly, such a reduced form approach would not be precise in capturing how certain financial market characteristics affect contracting efficiency. In order to provide more insight and to be more specific on what determines an agent's hedging ability, we adapt a search-theoretic framework.

While it is admittedly an abstract framework, the search model we propose enables us to introduce two notions that seem to be practically relevant. In practice, a manager trying to build a portfolio to diversify his firm-specific risk is likely to choose from a variety of financial instruments. Accordingly, one notion that affects the agent's portfolio problem is the extent of the asset variety in the financial market, i.e., the financial market sophistication. A related issue that arises as a result of variety is informational. Since the risk that the agent tries to diversify is by its nature firm-specific, it is unlikely that the agent will have ex ante full knowledge of how all the different assets/portfolios are correlated with this particular risk. This consideration introduces the second notion we would like to explore in endogenizing the agent's diversification ability: the cost of acquiring information on the statistical characteristics of different portfolios available.[16] The search framework we analyze below captures both of these notions, namely the financial market sophistication and cost of information acquisition in the market, as two determinants of an agent's diversification ability.

3.2 Search for a Portfolio

The analysis of the previous section establishes that given e^* and d^* an agent's expected utility increases in the quality of the portfolio he holds. This expected utility is taken with respect to $\mathbf{Q}$, conditional on his search strategy. In this section, we characterize the optimal search strategy of an agent in the financial market who holds a contract (s,t). This analysis endogenizes the portfolio quality z that the agent holds and hence the amount of compensation risk that he diversifies in the financial market.

Formally, the search process is modeled as follows: Let $\mathbf{Q}_1, \mathbf{Q}_2, \cdots$ be an i.i.d. sequence of random variables with an absolutely continuous distribution function F; we denote the density function by f. The agent can observe the sequence $\mathbf{Q}_1, \mathbf{Q}_2, \cdots$ at a unit cost $\kappa > 0$ as long as he wishes.[17] For each $n = 1, 2, \cdots$ after observing

[16] As mentioned, the information that the agent needs to effectively diversify contractual risks is, by its very nature, agent-specific: it is not some information of public interest, like the expected return of a financial asset. As such, it seems unlikely that this piece of information is publicly available in the market, nor can it be inferred easily from the asset prices. This informational obstacle for diversification has also been emphasized by Allen and Gale (2000) who argue that "[...] it is easy to imagine situations in which different agents are interested in quite different kinds of information. Suppose that agents want to hedge some idiosyncratic risks, for example, their future incomes from employment. Then each agent will want to know the correlation between the return to a particular asset and his own labor income." (p.449)

[17] Note that we study the problem as if the agent has no memory, in the sense that he does not have access to the realizations observed in the past. However, this assumption is only for expositional ease: analysis of the case where the agent perfectly recalls the past is exactly the same. To see this, simply observe that if the agent's optimal strategy does not require him to stop after a realization q_n, he will not

$\mathbf{Q}_1 = q_1, \mathbf{Q}_2 = q_2, \cdots, \mathbf{Q}_n = q_n$ the agent can stop and enjoy the utility

$$-\exp\Big\{-a\Big(sf(e^*) + t - c(e^*) - \frac{a}{2}s^2(\sigma_\epsilon^2 - q_n^2) - \kappa n\Big)\Big\},$$

or he may continue and observe $\mathbf{Q}_{n+1}$.

To formally state the agent's search problem, let us write $\beta := \exp\{a\kappa\}$, and define the random variable

$$\mathbf{U}_n := -\exp\Big\{-a\Big(sf(e^*) + t - c(e^*) - \frac{a}{2}s^2(\sigma_\epsilon^2 - \mathbf{Q}_n^2)\Big)\Big\}.$$

DEFINITION 1 *A* stopping rule *is a function* $\sigma : \bigcup_{\tau=1}^{\infty} \mathbb{R}^\tau \mapsto \{0,1\}$*, such that after the observation* $\mathbf{q}_n = (q_1, q_2, \cdots, q_n)$*, the agents stops if* $\sigma(\mathbf{q}_n) = 0$*, and continues sampling if* $\sigma(\mathbf{q}_n) = 1$.

The agent's problem is to find a stopping rule $\sigma \in \Sigma$ *that maximizes the expected utility*

$$\mathsf{E}(\beta^n \mathbf{U}_n), \tag{3}$$

where Σ *is the set of all stopping rules.*

For any stopping rule $\sigma \in \Sigma$, let $v(\sigma)$ denote the value of the expected utility given in (3) and define $v^* = \sup_{\sigma\in\Sigma} v(\sigma)$. We first establish the existence of the stopping rule.

PROPOSITION 2 *For the search problem in Definition 1, there exists an optimal stopping rule* $\sigma^* \in \Sigma$ *such that* $v(\sigma^*) = v^*$.

PROOF If $\mathsf{E}(|\mathbf{U}_n|) < \infty$, and $lim_{n\to\infty}\beta^n\mathbf{U}_n = -\infty$, then the existence of stopping rule is standard (see DeGroot (1970).) Let us write $\mathbf{U}_n := \psi \exp\{\varphi \mathbf{Q}_n^2\}$ where $\psi = -\exp\big\{-a\big(sf(e^*) + t - c(e^*) - \frac{a}{2}s^2\sigma_\epsilon^2\big)\big\}$ and $\varphi = -\frac{a^2}{2}s^2$. Observe that $\mathsf{E}(|\mathbf{U}_n|) = |\psi| \int \exp\{\varphi t^2\} dF(t)$. Because $\varphi < 0$ we immediately get $\mathsf{E}(|\mathbf{U}_n|) < \infty$. $lim_{n\to\infty}\beta^n\mathbf{U}_n = -\infty$ is obvious. Q.E.D.

At any given stage of the search, the agent knows that if he continues sampling, the optimal stopping rule σ^* yields the value v^*. Therefore, at the continuation, the agent pays the sampling cost and gets the maximum of βv^* and the realization of $\beta\mathbf{U}$. This means that the continuation value should be equal to the expectation of the maximum of βv^* and $\beta\mathbf{U}$. Hence, the optimality equation of the agent's search problem is

$$v^* = \beta\mathsf{E}\Big(\max\{\mathbf{U}, v^*\}\Big). \tag{4}$$

stop at any future realization smaller than q_n, because of the i.i.d. nature of the process. Therefore, the agent will keep searching only because he aims to improve what he currently holds.

The agent stops searching when the realization q yields a utility (excluding the sampling cost) larger than v^*:

$$v^* \leq -\exp\Big\{-a\Big(sf(e^*)+t-c(e^*)-\frac{a}{2}s^2(\sigma_\epsilon^2-q^2)\Big)\Big\}.$$

Rearranging and solving for q, one observes that the agent's optimal search strategy is a cutoff rule with the threshold $z^* := (q^*)^2$. The agent stops sampling if and only if

$$q^2 \geq z^* := \sigma_\epsilon^2 - \frac{2}{as^2}\left(sf(e^*)+t-c(e^*)+\frac{1}{a}\ln(-v^*)\right). \tag{5}$$

The agent continues sampling until he finds a portfolio with quality q^* so that he can reduce the variance of his wealth distribution to $s^2(\sigma_\epsilon^2 - z^*)$. In particular, the agent's optimal search procedure is the threshold strategy: if the realization of $\mathbf{Q}$ is between $-|q^*|$ and $|q^*|$, then the agent keeps searching, and he stops when $-|q^*| \geq q$ or $q \geq |q^*|$. The following Proposition characterizes the threshold portfolio quality q^*.

Proposition 3 *For a given compensation contract (s,t), the agent's optimal stopping rule σ^* in the financial market is characterized by a pair (ν, q^*) such that*

$$\begin{aligned}\beta^{-1} - \big[F(q^*)-F(-q^*)\big] &= \int^{-q^*} \exp\Big\{-\frac{a^2s^2}{2}(\tau^2 - z^*)\Big\}dF(\tau) \\ &\quad + \int_{q^*} \exp\Big\{-\frac{a^2s^2}{2}(\tau^2 - z^*)\Big\}dF(\tau)\end{aligned} \tag{6}$$

and the agent stops at

$$\nu = \min\big\{n \in \{1,2,\cdots\} : q_n^2 \geq z^*\big\}.$$

Proof See Appendix. Q.E.D.

Using the optimal stopping rule in Proposition 3, one can immediately link the intensity of the agent's search behavior in the financial market to the pay-performance sensitivity, s, of his contract and the unit search cost, κ. Consider first how an increase in s affects the agent's optimal search behavior. A contract with a higher pay-performance sensitivity exposes the agent to more risk, and increases the variance of his wealth distribution. As a result, the agent's demand for diversification and the threshold portfolio quality he is content with increases, causing the agent to search more aggressively, looking for a portfolio with a higher correlation with his contractual risk.

Proposition 4 *Suppose f is differentiable. As the agent's pay-performance sensitivity s increases, he searches more aggressively in the financial market. That is, $dz^*(s)/ds \geq 0$: as s increases, the optimal stopping threshold z^* increases.*

PROOF See Appendix. Q.E.D.

Now consider the effect of the agent's unit cost, κ, of search on the optimal search behavior. This search cost can be interpreted as the agent's cost of information acquisition in the financial market: by incurring κ, the agent learns the correlation of a portfolio with his contractual risk and hence how well this portfolio fits his diversification purposes. Not surprisingly, as it becomes more costly to acquire this information, the agent becomes content with an asset with a lower correlation; i.e., he searches less aggressively.[18]

PROPOSITION 5 *Suppose f is differentiable. As the search cost κ increases, the agent searches less aggressively in the asset market. That is, $dz^*(s)/d\kappa \leq 0$: as κ increases, the optimal stopping threshold z^* decreases.*

PROOF See Appendix. Q.E.D.

Clearly, the extent to which a wide variety of financial portfolios are available in the market affects the agent's optimal search behavior. To be able to build a link between the intensity of the agent's search and the availability of portfolios correlated with his contractual risk, we now introduce a simple concept of financial market sophistication.

FINANCIAL MARKET SOPHISTICATION. The other important characteristic of the financial market is captured by the distribution of the random variable $\mathbf{Q}$. Notice that F describes the availability of different portfolios in the market and measures the market's "sophistication." Let us illustrate with a simple example: let u_x denote the uniform density function with support $[-x, x]$. Consider u_a and u_b for $a > b$ and let us refer to them as u_a- and u_b-market respectively. Because $[-b, b] \subset [-a, a]$, there are portfolios available in u_a-market that are not available in u_b-market. In other words, u_a-market is larger than the u_b-market for the agent's diversification purposes. Obviously, comparing the support of different markets alone is not sufficient to define "market sophistication." It may simply be that although one market is larger than the other, the former has too little weight on the tails, so that it is costly to attain them. To capture these ideas, we restrict our attention to a class of markets (pdfs) ordered according to a well-known relation: *mean-preserving spread*. We refer to a market as more sophisticated than another if the former's density function is a mean-preserving spread of the latter's. We write $g \succeq f$ when g is a mean-preserving spread of f.

Our goal is to show that an agent searches more aggressively in the g -market than in the f-market if $g \succeq f$. In fact our simple example immediately provides the intuition.

[18]Our exercise is akin to random search models in labor economic and similar results are obtained in the theory of job search. In particular, the notion of reservation wage is analogous to the threshold quality of a portfolio and they react similarly to changes in search cost. For a general discussion of job search models see Cahuc and Zylberberg (2004), and Devine and Kiefer (1991).

We immediately observe $u_a \succeq u_b$. An agent in the u_a-market can easily restrict his search in $[-b, b]$ and reach a utility at least as good as what he can get in the u_b-market. It is easy to verify our claim and to see the intuition for this little example. In what follows, we focus on a large class of markets and therein prove our claim.

Definition 2 *Let $\mathfrak{F} := \{f_\alpha\}$ be a family of distribution functions indexed by α such that (i) $\int t f_\alpha(t)dt = 0$ for any α and $f(t) = f(-t)$ for all t, (ii) $f_{\alpha'} \succeq f_\alpha$ for any $\alpha' > \alpha$, (iii) for any α, α' , f_α and $f_{\alpha'}$ satisfy the following single-crossing property in $\mathbb{R}_+$: if $f_\alpha(x) - f_{\alpha'}(x) = 0$ and $f_\alpha(x') - f_{\alpha'}(x') = 0$ for some $x' > x > 0$ then $f_\alpha(y) - f_{\alpha'}(y) = 0$ for any $y \in [x, x']$.*

Let us briefly discuss the assumptions. We first restrict our attention to a class of density functions that are symmetric around zero. Although this seems like a technical restriction, it is merely an assumption in terms of the qualitative aspect of the problem at hand: from the viewpoint of our agent, what really matters is $|q|$. The second assumption restricts our analysis to a class of functions such that we can compare any two distribution functions with respect to the relation $\succeq$ that is central to the idea of market sophistication. Finally, for technical reasons we assume the single-crossing condition. Although it narrows the class of function we work with, we still have many well-known distributions that satisfy these assumptions. The family of normal distributions with zero mean and variance α is just one example. For proper choice of parameters, uniform, logistic, cauchy, student's t, and beta distributions also satisfy the assumptions.

Note that for $f \in \mathfrak{F}$ we can re-write equation (6) as

$$1 - \beta^{-1} = 2\int_{q^*} 1 - \exp\Big\{ -\frac{a^2 s^2}{2}\Big(\tau^2 - z^*\Big)\Big\} dF(\tau).$$

Now for any $f \in \mathfrak{F}$ define

$$\varphi(\tau; f) := \int_\tau h(t) f(t) dt$$

where $h(t) := 1 - \exp\Big\{ -\frac{a^2 s^2}{2}(t^2 - \tau^2)\Big\}$.

Lemma 1 *For any $f \in \mathfrak{F}$, $\varphi(\tau; f)$ is non-increasing in τ. Furthermore for any $f, g \in \mathfrak{F}$, if $g \succeq f$ then $\varphi(\tau; f) \leq \varphi(\tau; g)$.*

Proof Let $f \in \mathfrak{F}$. It is straightforward to see that

$$\frac{d\varphi(\tau; f)}{d\tau} = -a^2 s^2 \int_\tau \big(1 - h(t)\big) dF(t) < 0.$$

Let $f, g \in \mathfrak{F}$, and assume that g is a mean-preserving spread of f. Also define $\iota := g - f$. We want to show that

$$\int_\tau h(t)\iota(t)dt \geq 0.$$

Note that $h(t) > 0$ for all $t \in \mathbb{R}$. Also, recall that g and f satisfy the single-crossing property. Because g is a mean-preserving spread of f, there exists at most one interval $[x_*, x^*]$ such that for any $x < x_*$ $\iota(x) < 0$ and for any $x > x^*$ $\iota(x) > 0$. The problem is trivial if $\iota(x) \geq 0$ for all $x \in \mathbb{R}_+$. Let $x_*, x^* \in \mathbb{R}_+$ such that $\iota(x_*) = \iota(x^*) = 0$. Denote $h_* := h(x_*)$ and $h^* := h(x^*)$. Now observe that $h(x)\iota(x) \geq h_*\iota(x)$ for all $x < x_*$ and $h(x)\iota(x) \geq h^*\iota(x)$ for all $x > x^*$ because h is monotonically increasing. Therefore, we have

$$\int_\tau h(t)\iota(t)dt \geq h^* \int_\tau \iota(t)dt \geq 0.$$

Hence, we get $\varphi(\tau; f) \leq \varphi(\tau; g)$ Q.E.D.

Corollary 1 *Let $f, g \in \mathfrak{F}$. There exists $\tau, \varsigma \in \mathbb{R}$ such that $\varphi(\tau; f) = 1/2(1 - \beta^{-1}) = \varphi(\varsigma; g)$. If $g \succeq f$, then $\varsigma \geq \tau$.*

Proof Let $f, g \in \mathfrak{F}$ and $\tau \in \mathbb{R}_{++}$ such that $\varphi(\tau; f) = 1/2(1 - \beta^{-1})$. Suppose f second order stochastically dominates g. Then by Lemma 1, $\varphi(\tau; g) \geq 1/2(1 - \beta^{-1})$, since $\varphi(\cdot; g)$ is non-increasing, $\varphi(\varsigma; g) = 1/2(1 - \beta^{-1})$ implies $\varsigma \geq \tau$. Q.E.D.

Under the light of these observations we are ready to state the result:

Proposition 6 *Suppose that there are two financial markets that are characterized by $f, g \in \mathfrak{F}$, respectively. If g is more sophisticated than f, the agent searches more aggressively in the g-market than the f-market. That is, the optimal stopping threshold in the g-market is larger than the threshold in the f-market.*

Proof The result follows from Corollary 1. Q.E.D.

3.3 Optimal Linear Incentive Scheme

We now analyze the principal's problem of setting the optimal linear compensation contract. In doing so, the principal takes into account the agent's optimal search for a portfolio, his subsequent optimal position in the portfolio and the optimal effort choice, e^*. Formally, the principal chooses the linear contract (s, t) to maximize

$$(1 - s)\mathsf{E}(\mathbf{X}_{e^*}) - t$$

subject to the participation constraint

$$\mathsf{E}(\beta^{\mathbf{N}_{v^*}})v^* \geq \bar{v},$$

where v^* is the optimal utility attained in the search process defined in equation (4), $\bar{v} = -\exp\{-a\bar{w}\}$ is the agent's reservation utility, and $\mathbf{N}_{v^*}$ is the random variable determining the number of times the agent has to search in order to reach v^*. Hence the term $\mathsf{E}(\beta^{\mathbf{N}_{v^*}}) =: \phi(z^*)$ is the *expected cost of optimal search*. We can write this participation constraint in terms of certainty equivalent wealth as

$$sf\big(e^*(s)\big) + t - c\big(e^*(s)\big) - \frac{a}{2}s^2\big(\sigma_\epsilon^2 - z^*(s)\big) - \bar{w} - \frac{1}{a}\ln\big(\phi(z^*)\big) \geq 0.$$

In the above expression the term $1/a \ln\big(\phi(z^*)\big)$ is the ex ante search cost of agent (measured in terms of wealth) to perform optimal search with threshold z^*. In equilibrium, the participation constraint holds as an equality. Solving for t and substituting it into the principal's objective function, the problem becomes choosing the agent's pay-performance sensitivity s to maximize the net surplus

$$f\big(e^*(s)\big) - c\big(e^*(s)\big) - \frac{a}{2}s^2\big(\sigma_\epsilon^2 - z^*(s)\big) - \bar{w} - \frac{1}{a}\ln\big(\phi(z^*)\big). \tag{7}$$

Differentiating (7) with respect to s, and using the optimality condition for e^* from (1) we obtain the following result:

PROPOSITION 7 *The optimal pay-performance sensitivity s^* satisfies one of the following three conditions:*

$$s^* \in (0, \bar{s}] \text{ and } (1-s)f'(e^*)\frac{de^*}{ds} = as\left(\sigma_\epsilon^2 - z^* - \frac{s}{2}\frac{dz^*}{ds}\right) + \frac{1}{a}\frac{\phi'(z^*(s))}{\phi(z^*(s))}\frac{dz^*(s)}{ds},$$
$$s^* \in (\bar{s}, 1) \text{ and } as\left(\sigma_\epsilon^2 - z^*(s) - \frac{s}{2}\frac{dz^*}{ds}\right) + \frac{1}{a}\frac{\phi'(z^*(s))}{\phi(z^*(s))}\frac{dz^*(s)}{ds} = 0,$$
$$s^* = 1 \text{ and } a\left(\sigma_\epsilon^2 - z^*(1) - \frac{1}{2}\left.\frac{dz^*}{ds}\right|_{s=1}\right) + \frac{1}{a}\frac{\phi'(z^*(1))}{\phi(z^*(1))}\left.\frac{dz^*(s)}{ds}\right|_{s=1} \leq 0,$$

where $\bar{s} = \min\{\tilde{s}, 1\}$ such that $e^*(\tilde{s}) = \bar{e}$. Moreover, s^* is non-increasing in the agent's search cost κ in the asset market and non-decreasing in the financial market's sophistication.

The first condition characterizes the interior solution when $e^*(s^*) \in (\underline{e}, \bar{e})$. In the second condition the agent's optimal effort for s^* exceeds the maximum effort $\bar{e}$ while s^* is still in the interior of $(0,1)$. In that case, although the incentive provided for the agent does not lead to a marginal increase in the effort level, it make the agent search more aggressively. Hence, at $e^* = \bar{e}$, the optimal choice of s equates the marginal decrease in the risk with the marginal increase in expected cost of search. Finally, the last condition refers to the boundary case in which the objective function is increasing at $s = 1$.

The optimal incentive scheme s^* in Proposition 7 equates the marginal benefit of incentive contracting through inducing higher effort to the marginal cost of exposing the agent to risk in order to elicit a given effort level. The optimal incentive scheme therefore is determined by the trade-off between providing effort incentives by making pay more sensitive to output, and compensating the risk-averse agent for bearing the associated risk exposure (insurance). In the current framework, the principal takes *into account the effect of s on the agent's subsequent search for a portfolio with quality $z^*(s)$ to diversify that risk*. When the agent can partially diversify the risk, he demands a lower risk premium for a given risk exposure. Accordingly, the financial market characteristics that increase the agent's diversification ability, and hence increase z^*, lower the principal's insurance costs.

As the search cost κ decreases, and/or as the financial market becomes more sophisticated, offering a wider variety of portfolios the agent can diversify more of his risk. As a result, the incentive consideration becomes more important when compared to the insurance consideration in determining the optimal compensation: the optimal pay structure shifts towards more performance-sensitive pay and less fixed pay as the cost of information acquisition in the financial market decreases and as the market becomes more sophisticated. In the limit as the search cost becomes arbitrarily small the agent hedges the entire risk (Proposition 5) if the financial market is sophisticated enough and first-best effort can be attained.

3.4 Both Principal and Agent can Search

In this subsection, we explore the possibility that the principal can also engage in a costly search of a portfolio and use that portfolio as part of the initial contract.[19] Suppose that the search technology available to principal is the same as we discussed in Section 3.2, except for the search cost. We denote the principal's unit search cost k. To differentiate the search problem of the principal let us denote the realization of the quality of a portfolio for him by $\tilde{q}$ and $\tilde{q}^2 =: \zeta$.

The problem of the principal is as follows. For each $n = 1, 2, \cdots$, after observing $\mathbf{Q}_1 = \tilde{q}_1, \mathbf{Q}_2 = \tilde{q}_2, \cdots, \mathbf{Q}_n = \tilde{q}_n$ the principal can stop and offer a contract (s, α, t) of the form $s\mathbf{X}_e + \alpha\mathbf{Y} + t$ where $\mathbf{Y}$ is the portfolio with quality $\tilde{q}_n^2$, or pay cost k and observe $\mathbf{Q}_{n+1}$.

When the principal stops and offers a contract, in the subgame that follows, the agent decides whether to accept the contract or not. If he accepts, the agent can also do further search and finally he makes the optimal effort choice. It is important to clarify

[19]We appreciate the referees' comments and questions that led us to explore this extension.

that further search by the agent does not mean he will *append* to the portfolio he is already offered in his contract, but rather he will *replace* it. Indeed, this is a plausible interpretation since we assume that each sampling in the search aims to improve the portfolio the agent—or the principal—builds during the search. Therefore, if the initial contract specifies s and an optimal position in a portfolio with quality ζ, the optimal quality the agent can obtain in the search should be $\max\{\zeta, z^*(s)\}$, for z^* defined in Propositions 3. Clearly, this means that if the agent accepts the contract he may engage in further search only when $\zeta < z^*$. An immediate implication of this observation is that in any contract that incentivizes the agent to do further search, the principal never finds it optimal to search and provide a portfolio as part of the initial contract. In order to see that, suppose the principal offers a contract that includes a portfolio with quality $\zeta < z^*(s)$. Let $\rho(\zeta)$ denote the principal's expected search cost with the stopping threshold ζ. Then the principal's problem is to maximize $(1-s)\mathsf{E}(\mathbf{X}_{e^*}) - t - \rho(\zeta)$ subject to the participation constraint $\phi(z^*)v^* \geq \bar{v}$. When we write the constraint in terms of certainty equivalent wealth and rearrange the objective function, the principal's problem is to maximize

$$f\big(e^*(s)\big) - c\big(e^*(s)\big) - \frac{a}{2}s^2\big(\sigma_\varepsilon^2 - z^*(s)\big) - \bar{w} - \frac{1}{a}\ln\big(\phi(z^*(s))\big) - \rho(\zeta).$$

Since it is clearly suboptimal to incur $\rho(\zeta)$ and compensate the agent for his expected optimal search cost by $\frac{1}{a}\ln\big(\phi(z^*(s))\big)$, in any optimal contract the principal either offers $\zeta \geq z^*$ or he completely lets the agent to do the search.

Having ruled out the possibility of an equilibrium contract in which both the principal and the agent undertake search, rather than characterizing the optimal contract, we will compare two problems to address the question of which party does the search in an optimal contract. While the first problem assumes that only the agent does the search, in the second problem only the principal undertakes the search. The next two objective functions respectively correspond to these problems.

$$f\big(e^*(s)\big) - c\big(e^*(s)\big) - \frac{a}{2}s^2\big(\sigma_\varepsilon^2 - z^*(s)\big) - \bar{w} - \frac{1}{a}\ln\big(\phi(z^*(s))\big),$$
$$f\big(e^*(s)\big) - c\big(e^*(s)\big) - \frac{a}{2}s^2\big(\sigma_\varepsilon^2 - \zeta\big) - \bar{w} - \rho(\zeta).$$

As we observed earlier in our analysis as κ becomes smaller $z^*(s)$ increases and the first-best effort is attained. This observation suggests that if the search cost for the agent is different than the principal's search cost, the answer of our question depends on the relative search cost. In particular, if the search cost, κ, of the agent is small enough compared to the principal's cost, k, then the principal may find it optimal to let the agent do the search. Although the exact comparison of the two problems with different search cost requires full characterization of principal's search problem that we described earlier, we can analyze the interesting case where the search costs are identical, $\kappa = k$. In order to tackle this question suppose that the *optimal* contract specifies pay-performance sensitivity $\hat{s}$. If the contract makes the agent do the search, the threshold would be $z^*(\hat{s})$. Alternatively, if the contract makes the principal do the search, the principal would specify a portfolio with quality, say, $\hat{\zeta}$. We already know that $\hat{\zeta} \geq z^*(\hat{s})$. Now, we claim that when the search costs are equal, the principal always prefers to undertake the search himself. To show that, let us consider the case where $\hat{\zeta} = z^*(\hat{s})$. Apparently, if the principal prefers to do the search himself when $\hat{\zeta} = z^*(\hat{s})$, in an optimal contract he would continue to do so even if $\hat{\zeta} > z^*(\hat{s})$.

The only difference between the two problems is whether the principal does the search and incur an expected cost of $\rho(\hat{\zeta})$ or lets the agent do the search by offering the agent an additional expected search cost compensation $(1/a \ln(\phi(z^*(\hat{s}))))$. We prove our claim in the next Proposition: when the search costs of the two parties are the same, for the same portfolio quality threshold the principal always prefers to undertake the search himself, rather than letting the agent do the search and compensate the agent for his search cost.

PROPOSITION 8 *Let z be a stopping threshold for the stopping problem stated in Section 3.2. Denote $\Delta(z) = F(z) - F(-z)$, and assume that $\Delta(z)e^{a\kappa} < 1$. The (expected) utility cost of search with a stopping threshold z for the agent and for the principal are*

$$\phi(z) := \frac{e^{a\kappa}\big(1-\Delta(z)\big)}{1-e^{a\kappa}\Delta(z)}, \qquad \rho(z) := \frac{k}{1-\Delta(z)},$$

respectively.

Suppose that $\kappa = k$. Compensating the agent in order to give incentives to him to undertake portfolio search is more costly for the principal than his own expected search cost if he undertakes the same search herself. That is $\frac{1}{a}\ln\big(\phi(z)\big) > \rho(z)$ for any z.

PROOF See Appendix. Q.E.D.

Let us summarize the main insight of this analysis. The agent's hedge trades in correlated portfolios are completely desirable for the principal. If the principal could trade financial portfolios that are correlated with firm-specific risk, then it would be optimal to tie agent's initial compensation to these financial portfolios to reduce the risk of the compensation scheme. If both the agent and the principal have access to financial markets to customize a hedge portfolio, the party with cheaper access (measured by search cost) to the market, but not both parties, should undertake the search. In that respect, the main result of this paper is to illustrate that there may be efficiency benefits associated with using financial markets to hedge firm-specific risks in compensation contracts: the precise optimal implementation of this hedging (whether the agent or the principal do the hedging) depends on which party has lower cost of unveiling and executing these portfolio opportunities. This point is similar to the insight provided by Garvey and Milbourn (2003) who argue that there is very little evidence that firms index executive compensation to aggregate market variables (relative performance evaluation) to remove marketwide risks from compensation schemes. They point out that executives can trade market indexes themselves, and hence the lack of relative performance evaluation might be explained by an executive's own ability to remove marketwide risks by trading in the financial markets. Using an executive's wealth and age as a proxy for the executive's cost of hedging by using a market index, they find strong empirical evidence of relative performance evaluation (market indexation) for younger executives. They conclude that firms should provide less relative performance evaluation as the cost to the manager of hedging on her own account decreases.

4 Discussion

The main insight of our analysis is that in a principal-agent framework the availability financial portfolios correlated with firm-specific risk can improve contracting efficiency by reducing the randomness in the risk averse agent's compensation scheme. While making this point, we contrast our positive result to existing negative results in the literature that show that an agent's hedging transactions in the financial markets typically undermine incentives. As we argued in Section 3, previous papers have considered settings in which the agent can trade financial instruments based on his own firm value: such trades undermine incentives since they reduce the sensitivity of agent's wealth to firm performance. In this paper, we illustrate a different type of hedging transaction: portfolios correlated with firm specific risk serve to diversify compensation risk while preserving the link between the agent's performance and his wealth. As a result, they reduce the insurance cost of incentive provision and enable the principal to elicit a higher level of effort. We endogenize the level of diversification that can be achieved in the financial market in a search theoretic framework. Our analysis indicates that—controlling for the intensity of the incentive problem and the level of firm-level risk—a financial hedge market with lower information-acquisition costs and higher sophistication (more portfolio variety) can result in a compensation contract with a higher pay-performance sensitivity and higher effort. This positive incentive implication is especially interesting given the strongly negative view on the emergence of managerial hedge markets in the U.S in the late 1990s. It is widely argued that the only possible function these hedge markets is to undermine the performance incentives in compensation schemes. [20]

As we argued, the crucial reason that our framework delivers a positive result is the particular hedging transaction we describe. In contrast to a hedging transaction contingent on the firm's performance (like an equity swap), the availability of a financial portfolio correlated with firm-specific risk improves contracting efficiency. It seems important to discuss more specifically how such an efficiency enhancing portfolio might be constructed to hedge the idiosyncratic risk in the compensation contract.

In order to be more specific, consider the CEO of a gold mining corporation who holds his own company stock as part of his compensation scheme. At the expense of oversimplification, but for the sake of a simple argument, let us assume that the company's revenues, and hence stock price are subject to gold price risk (which is common to all gold mining companies) and production/output risk specific to this company. More concretely, suppose that this gold company is heavily engaged in exploring and developing gold mines in a Latin American country that recently elected a left-oriented populist government with a nationalization agenda that may target foreign companies. If this company's main existing and future gold production ability is heavily dependent on the mines they currently operate in that particular country, one would imagine that a high proportion of this company's firm-specific production risk stems from this nationalization threat.

The CEO, or the shareholders who design the compensation scheme, can hedge the CEO's gold price risk with a long position in U.S. dollar index, which is typically negatively correlated with gold prices. This type of transaction would hedge an industry specific risk factor in the compensation scheme, namely the gold price, and would be akin to the type of hedging considered in Jin (2002), Acharya and Bisin (2005) and Özertürk (2006). However, in terms of hedging against a possible plunge in production capacity due to nationalization, which constitutes the main firm-specific risk in our example, a long position in the U.S. dollar index would not be helpful at all. Consider then an investment portfolio that tracks foreign investment flows to the country in question. The value of this investment portfolio would be negatively correlated with the nationalization efforts of the government: to the extent that the nationalization agenda is carried out, foreign investment flow would decline. In that respect, a short position in a portfolio that tracks foreign investment flows to the country in our example would work to diversify some of the company's firm-specific production risk. One might argue that constructing such an investment vehicle might be cumbersome, and furthermore the extent that such a hedge works depends on the degree of correlation between company's

[20] For example, an editorial in *The Economist*, ('Executive Relief' April 3, 1999, p.64) states that "...Further justifying the scepticism is the current popularity of derivatives that allow managers to hedge their exposure to their own company's shares. [...] Such hedging is wholly against the spirit of the massive awards of shares and share options." (See also Lavalle (2001).

production risk in that country and how the talk of nationalization affects foreign investment flows. We should point out that this is precisely the point we aim to make by our costly search of a portfolio framework. Unlike instruments that are useful in hedging industry-wide aggregate risks (like gold price risk in our example), financial instruments that can help to hedge idiosyncratic risk are much less readily available. Successful hedging of firm-specific risk requires identifying the singular adverse scenarios a company may face (i.e., what constitutes firm-specific risk) and seeking out investment vehicles that are correlated with the possibility of such adverse events.

Inefficient Corporate Risk Management One relatively less emphasized implication of tying the agent/manager's compensation to firm value is that it may provide the agent with inefficient risk reduction incentives in his technology choice, merely to reduce the risk embedded in his compensation contract.[21] For example, the manager may respond to a higher pay-performance sensitivity by choosing to avoid risky projects with positive net present values (as in Lambert (1986)) or, similarly, he may undertake inefficient asset acquisitions to lower the firm-specific risk (as in Amihud and Lev (1981)). Tufano (1996) examines corporate risk management activity in the North American gold mining industry and confirms that firms whose managers hold more stock based compensation manage more gold price risk. His study concludes that risk reduction policies may be set to satisfy the needs of poorly diversified managers, and do not necessarily maximize firm value. Our analysis suggests that the agent's ability to trade financial portfolios correlated with firm-specific risk would mitigate such inefficient risk reduction incentives. To the extent that the risk averse agent can diversify the firm-specific risk in his compensation contract by holding a position in a correlated portfolio, he has less appetite for inefficient risk reduction at the firm technology level.

5 Concluding Remarks

In this paper, we analyze a principal-agent model in which the agent can trade financial assets that are correlated with his contractual risk for the purposes of diversification before making his effort decision. Our key innovation is that the agent does not have full information about how different assets traded in the market fit his diversification purposes. Therefore, the agent engages in costly information acquisition in the asset market, which we model as a search process. By random sampling from the set of available assets, the agent can learn the correlation of a portfolio, and hence how well it fits his diversification purposes.

[21]Formally, this inefficient risk reduction would correspond to a model where the agent has the ability to lower σ_ε^2 directly even if that risk reduction at the technology level would reduce expected firm value as well, and hence would be undesirable from the perspective of risk neutral shareholders.

This search framework allows us to introduce the informational obstacles that an agent faces in diversifying contractual risks and endogenizes the amount of compensation risk that the agent diversifies. The agent searches more aggressively in the asset market and diversifies more risk: *1*) as the pay-performance sensitivity of his compensation contract increases; *2*) as the cost of information acquisition in the asset market (measured by the search cost) decreases; and *3*) as the sophistication of the asset market (measured by the variety of financial assets available) increases. Our framework introduces the cost of information and financial market sophistication into the principal-agent environment as two determinants of the optimal pay-performance sensitivity and contracting efficiency. We show that as the agent gains access to a financial market with lower information acquisition costs and higher sophistication, the optimal compensation contract involves more performance-sensitive pay and elicits a higher level of effort.

By allowing the agent to search for and trade financial assets correlated with the idiosyncratic risk in his contract, we emphasize the positive efficiency implications of such trades. A position in a portfolio correlated with the agent's idiosyncratic risk serves to reduce the randomness in his wealth while preserving the link between his wealth distribution and the subsequent effort choice. Accordingly, such a portfolio opportunity lowers the insurance cost of incentive contracting and improves contracting efficiency. This positive result complements the existing negative results that follow from models in which the agent can borrow and save, or trade side contracts contingent on his own output, thus undermining the link between his effort decision and his wealth distribution.

Appendix

Proof of Proposition 3

Proof Note that equation (4) can be written as

$$\begin{aligned} v^* &= \beta \int_{-q^*}^{q^*} v^* dF(t) + \beta \int^{-q^*} - \exp\big\{ - a(\varphi - \psi\sigma_\epsilon^2) - a\psi t^2 \big\} dF(t) + \beta \int_{q^*} - \exp\big\{ - a(\varphi - \psi\sigma_\epsilon^2) - a\psi t^2 \big\} dF(t) \\ &= \frac{\beta}{1 - \beta \Delta F^*} \left(\int^{-q^*} - \exp\big\{ - a(\varphi - \psi\sigma_\epsilon^2) - a\psi t^2 \big\} dF(t) + \int_{q^*} - \exp\big\{ - a(\varphi - \psi\sigma_\epsilon^2) - a\psi t^2 \big\} dF(t) \right) \end{aligned}$$

where $\Delta F^* = F(q^*) - F(-q^*)$. Expressing v^* in terms of z^* by use of (5), and rearranging yields the result.

$$\begin{aligned} - \exp\Big\{ - a\big(\varphi - \psi(\sigma_\epsilon^2 - z^*)\big) \Big\} &= \frac{\beta}{1 - \beta \Delta F^*} \left(\int^{-q^*} - \exp\big\{ - a(\varphi - \psi\sigma_\epsilon^2) - a\psi t^2 \big\} dF(t) \right. \\ &\quad \left. + \int_{q^*} - \exp\big\{ - a(\varphi - \psi\sigma_\epsilon^2) - a\psi t^2 \big\} dF(t) \right) \\ \frac{1 - \beta \Delta F^*}{\beta} &= \int^{-q^*} \exp\Big\{ - \frac{a^2 s^2}{2} (t^2 - z^*) \Big\} dF(t) \\ &\quad + \int_{q^*} \exp\Big\{ - \frac{a^2 s^2}{2} (t^2 - z^*) \Big\} dF(t). \end{aligned} \tag{8}$$

Q.E.D.

Proof of Proposition 4

Proof Taking the derivatives of both sides of (8) with respect to s, we get:

$$\begin{aligned} -\big(f(q^*) + f(-q^*)\big) \frac{dq^*}{ds} &= \int^{-q^*} \left(\frac{a^2 s^2}{2} \frac{dz^*}{ds} - a^2 s (t^2 - z^*) \right) \exp\Big\{ - \frac{a^2 s^2}{2} (t^2 - z^*) \Big\} dF(t) \\ &\quad + \int_{q^*} \left(\frac{a^2 s^2}{2} \frac{dz^*}{ds} - a^2 s (t^2 - z^*) \right) \exp\Big\{ - \frac{a^2 s^2}{2} (t^2 - z^*) \Big\} dF(t) \\ &\quad - \big(f(q^*) + f(-q^*)\big) \frac{dq^*}{ds} \end{aligned}$$

Re-arranging yields the result:

$$\frac{dz^*}{ds} = \frac{2}{s} \frac{\int^{-q^*} (t^2 - z^*) \exp\big\{ - \frac{a^2 s^2}{2} (t^2 - z^*) \big\} dF(t) + \int_{q^*} (t^2 - z^*) \exp\big\{ - \frac{a^2 s^2}{2} (t^2 - z^*) \big\} dF(t)}{\int^{-q^*} \exp\big\{ - \frac{a^2 s^2}{2} (t^2 - z^*) \big\} dF(t) + \int_{q^*} \exp\big\{ - \frac{a^2 s^2}{2} (t^2 - z^*) \big\} dF(t)} > 0.$$

Q.E.D.

Proof of Proposition 5

Proof Taking the derivatives of both sides of (8) with respect to κ, we get:

$$\begin{aligned}
-a\exp\{-a\kappa\} - \frac{dz^*}{d\kappa}(f(q^*) + f(-q^*)) &= \frac{dz^*}{d\kappa}\int^{-q^*}\left(\frac{a^2s^2}{2}\right)\exp\Big\{-\frac{a^2s^2}{2}(t^2 - z^*)\Big\}dF(t) \\
&\quad + \frac{dz^*}{d\kappa}\int_{q^*}\left(\frac{a^2s^2}{2}\right)\exp\Big\{-\frac{a^2s^2}{2}(t^2 - z^*)\Big\}dF(t) \\
&\quad - \frac{dz^*}{d\kappa}(f(q^*) + f(-q^*))
\end{aligned}$$

Re-arranging yields the result:

$$\frac{dz^*}{d\kappa} = -\frac{2\exp\{-a\kappa\}}{as^2\left(\int^{-q^*}\exp\Big\{-\frac{a^2s^2}{2}(t^2 - z^*)\Big\}dF(t) + \int_{q^*}\exp\Big\{-\frac{a^2s^2}{2}(t^2 - z^*)\Big\}dF(t)\right)} < 0.$$

Q.E.D.

Proof of Proposition 8

Let z be the stopping threshold. For a given F let $\Delta(z) = F(z) - F(-z)$. Assume that $e^{a\kappa}\Delta(z) < 1$. The expected utility cost of search for the agent is

$$\begin{aligned}
\phi(z) &:= e^{a\kappa}(1 - \Delta(z)) + e^{2a\kappa}(1 - \Delta(z))\Delta(z) + e^{3a\kappa}(1 - \Delta(z))\Delta(z)^2 + \cdots \\
&= e^{a\kappa}(1 - \Delta(z))\left[1 + e^{a\kappa}\Delta(z) + e^{2a\kappa}\Delta(z)^2 + \cdots\right] \\
&= \frac{e^{a\kappa}(1 - \Delta(z))}{1 - e^{a\kappa}\Delta(z)}.
\end{aligned}$$

Obviously $\phi(z)$ is infinite if $e^{a\kappa}\Delta(z) \geq 1$. On the other hand, the expected utility cost of search for the principal is:

$$\begin{aligned}
\rho(z) &:= k(1 - \Delta(z)) + 2k(1 - \Delta(z))\Delta(z) + 3k(1 - \Delta(z))\Delta(z)^2 + \cdots \\
&= k(1 - \Delta(z))\left[1 + 2\Delta(z) + 3\Delta(z)^2 + \cdots\right] \\
&= k(1 - \Delta(z))\left[\frac{1}{1 - \Delta(z)} + \frac{\Delta(z)}{1 - \Delta(z)} + \frac{\Delta(z)^2}{1 - \Delta(z)} + \cdots\right] \\
&= \frac{k}{1 - \Delta(z)}.
\end{aligned}$$

Observe that the principal should pay the agent $\frac{1}{a}\ln(\phi(z))$ in order to compensate his expected utility cost. Therefore for a given z, in order to decide which one is higher for the principal we have to compare $\frac{1}{a}\ln(\phi(z))$ and $\varphi(z)$.

To show that it is more costly for the agent it will be enough to show that

$$\begin{aligned}
\frac{1}{a}\ln\left(\frac{e^{a\kappa}(1 - \Delta(z))}{1 - \Delta(z)e^{a\kappa}}\right) &> \frac{\kappa}{1 - \Delta(z)} \\
\frac{e^{a\kappa}(1 - \Delta(z))}{1 - \Delta(z)e^{a\kappa}} &> e^{\frac{a\kappa}{1 - \Delta(z)}} \\
(1 - \Delta(z)) + \Delta(z)e^{\frac{a\kappa}{1 - \Delta(z)}} - e^{a\kappa\frac{\Delta(z)}{1 - \Delta(z)}} &> 0.
\end{aligned}$$

Let us write $x = \Delta(z)$ and define $f(x) := 1 - x + xe^{\frac{a\kappa}{1-x}} - e^{\frac{a\kappa x}{1-x}}$. Observe that $f(0) = 0$ and

$$\begin{aligned} f'(x) &= -1 + e^{\frac{a\kappa}{1-x}} + \frac{a\kappa x}{(1-x)^2} e^{\frac{a\kappa}{1-x}} - \frac{a\kappa}{(1-x)^2} e^{\frac{a\kappa x}{1-x}} \\ &= -1 + e^{\frac{a\kappa}{1-x}} \left(1 + \frac{a\kappa x}{(1-x)^2} - \frac{a\kappa}{(1-x)^2} e^{-a\kappa} \right) \\ &\geq 0 \end{aligned}$$

since $xe^{a\kappa} < 1$. Therefore $f(x) > 0$ for all $x > 0$. Hence, for the same threshold, the principal always prefer to make the search himself, rather than letting the agent do the search and compensate him (through the participation constraint.)

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